

# Unit - I

## Data Analysis

In analytical chemistry we are going to get a lot of data. How to analyse and interpret these data? This we shall see in this chapter.

### 1. ERRORS IN CHEMICAL ANALYSIS

#### *Definition of Error :*

The difference between the measured value of a property and its accurate value is called the error.

#### *Explanation :*

During chemical analysis we measure the value of a particular property. E.g., we measure the weight of an object or the volume of a solution. Accurate results will be got when persons with great skill do the measurements with best instruments. This is nearly impossible. Usually the measured value of the property will never be the accurate value of the property. The difference between these two is called the error. Such errors in measurement will affect the accuracy and precision of the property that is measured. So, the analytical data so obtained becomes unreliable. In the following pages we shall study about such errors in chemical analysis.

#### **Classification of Errors :**

The errors that arise in a chemical analysis are classified into two types. They are

- i. **Determinate errors**
- ii. **Indeterminate errors (or) Random errors.**

It should be remembered that it is difficult or impossible to be certain of the type to which a given error belongs. Never the less the classification is useful for discussing analytical errors.

## **Determinate Errors (or) Systematic errors**

These are errors which have a definite value and an assignable cause. The analyst can measure and account for these errors. These can be avoided. They are unidirectional i.e., the errors will be either more or less than the accurate value. From this, they can be identified.

### *Sources of these errors :*

- i. Defective instruments
- ii. Careless operation
- iii. Procedural defects

### *Classification :*

- i. Instruments errors.
- ii. Method errors .
- iii. Personal errors.

#### **i. Instrument Errors :**

When we use balances, weights, pipettes, burettes etc., we must make sure that they are not defective. For example, a weight marked 10g may not be 10g after all. So to avoid these errors one must use best instruments. Periodic calibration of apparatus and weights is a must. These errors may be identified by changing the instrument, the error will also change.

#### **ii. Methods Errors :**

These are introduced by defective experimental procedures. E.g.,

- a. Co - precipitation or post precipitation in gravimetric analysis.
- b. Usage of improper indicators in volumetric analysis.

These are difficult to identify. So these are the most serious of the three types of determinate errors. So to avoid these, one must be thorough with the theoretical part of the experiment.

#### **iii. Personal Errors (or) Operative Errors**

These are introduced by personal defects or carelessness. The Sources of this error are human defects in eyes, mind etc., In colorimetric

experiments errors will be introduced by a person who is colourblind. A person with defective eyes will invariably note readings erroneously. Carelessness, fatigue and improper instructions from the teacher also introduce these errors. Several mistakes may creep in E.g., Wrong calculations, wrong placement of decimals, noting wrong signs, cooking results etc. These can be avoided if one works scrupulously in the laboratory. These are identified by the fact that these errors change when the measurement is repeated.

Determinate errors may also be classified as being either

1. Constant Errors and 2. Proportional Errors.

### 1. Constant Errors :

These are errors whose magnitude is independent of the size of the sample taken for analysis. For example, let us say that 0.5 mg of precipitate is lost when washed with 200ml of the wash liquid. Now, if we wash 500mg of precipitate with 200 ml of wash liquid 0.5 mg of the precipitate will be lost. So the loss is  $(0.5 \times 100) \div 500 = 0.1\%$ . Let us assume that we wash 50mg of the precipitate with 200ml of wash liquid, here also 0.5mg will be lost. So the loss is  $(0.5 \times 100) \div 50 = 1\%$ .

Thus we find that a constant error will become more serious as the size of the quantity measured decreases. So to minimize the effect of constant error we have to use a large, sample. [Please remember that in our experiments in gravimetric analysis the solution to be estimate is so prepared that the weight of the precipitate is around 0.2g].

### 2. Proportional Errors :

These are errors whose magnitude increases or decreases in proportion to the size of the sample taken for analysis. Invariably impurities in the sample. If not removed, will cause a proportional error.

### Correction of Determinate Errors :

Determinate instrumental errors are corrected by calibrating the

instruments concerned. The equipments are to be calibrated periodically. This is because instrumental errors arise due to wear corrosion or mistreatment.

Determinate personal errors can be minimised by care and self discipline. Instrument readings, note book entries and calculations should be checked systematically.

Determinate method errors are particularly difficult to detect. They may be corrected by one or more of the following procedure.

**1. *Analysis of standard samples :***

A method may be tested for determinate error by analysis of a synthetic sample whose over all composition is known and which closely resembles the material to be tested by the particular method.

**2. *Independent analysis :***

When samples to be analysed are not available in a pure state this method is used. The sample is analysed in a particular method. Then it is analysed by a different method of established reliability.

**3. *Blank determinations :***

Constant errors affecting physical measurements can be frequently evaluated with a blank determination in which all steps of the analysis are performed in the absence of the sample. The result is then applied as a correction to the actual measurement. This method is useful to correct errors that the due to the introduction of interfering contaminants from reagents and vessels employed in the analysis. This method is useful to correct titration data in volumetric analysis.

**4. *By taking large sample size :***

We know that a constant error decreases as the size of the sample is increased. So, to correct such type of errors large sample size is used for analysis.)

**Random Errors or Indeterminate Errors or Accidental Errors**

They are errors arising from uncertainties in a measurement that are

unknown and not controlled by the person doing an experiment.

**Sources :**

- i. Instrument uncertainties
- ii. Method uncertainties
- iii. Personal uncertainties.

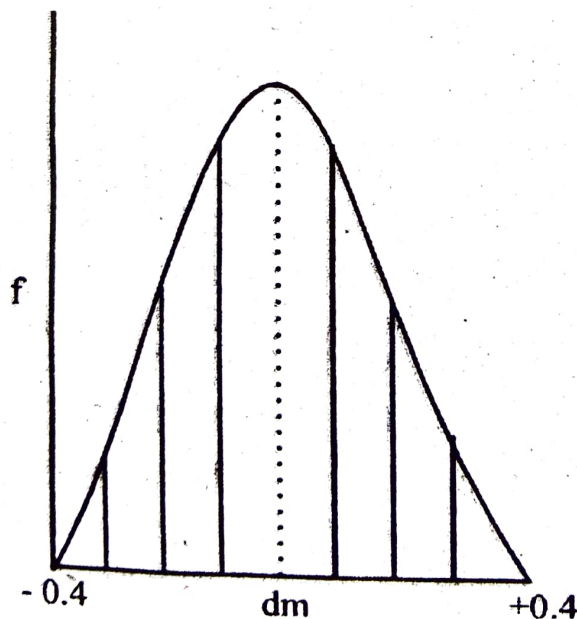
**Identification :**

Identification of indeterminate errors is difficult. Scatter of data about the mean is the effect of an indeterminate error.

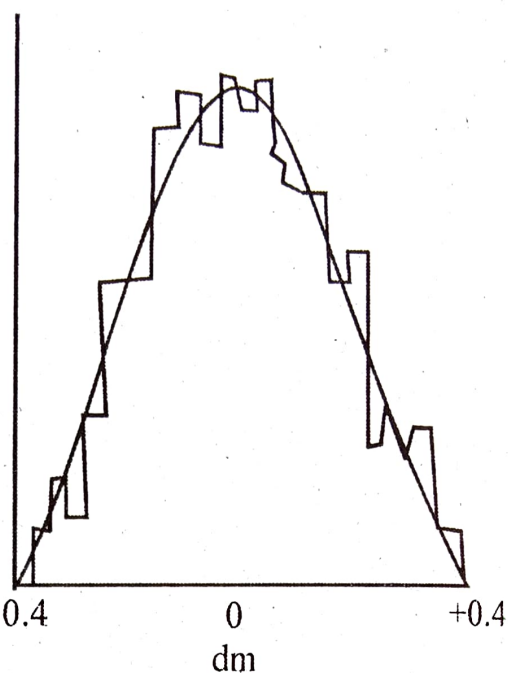
**Error Analysis :**

When indeterminate error or deviation from mean ( $dm$ ) is plotted against its frequency ( $f$ ) we get a curve as shown in figure. This bell shaped curve is called Gaussian or normal error curve. The properties of this normal curves are

- i. The frequency is maximum where the indeterminate error is nil.
- ii. There is a symmetry about this maximum, suggesting that positive and negative errors occur with equal frequency.
- iii. As the magnitude of the error increases, the frequency decreases exponentially.



In chemical analysis, indeterminate errors follow this Gaussian type distribution. For e.g., if the deviation from the mean ( $dm$ ) of hundreds of repetitive weighing measurements on a single object, are plotted against the frequency ( $f$ ) of occurrence of each deviation we get a curve as shown in figure. This curve proves the fact that a number of small independent and uncontrolled uncertainties are there in our normal measurements.



These uncertainties manifest in the result.

This Gaussian distribution of most analytical data, permits us to use statistical techniques to estimate the extent of indeterminate errors. Thus we use several statistical techniques like mean, median, average deviation, standard deviation etc., while analysing our experimental results.

### Minimising Errors :

From the above discussion it is clear that, if we want to minimise error we have to minimise determinate as well as indeterminate errors.

To minimise determinate errors we will have to use standard, internationally accepted instruments. The various measuring aids must be calibrated periodically and got certified according to international standards. All reagents must be properly maintained. Dependable procedures must be adopted to avoid methodic errors. To avoid personal errors one must be careful and honest in recording the observation. It is human tendency to manipulate results to get high degree of precision. This tendency must be avoided.

We know that indeterminate errors are uncontrollable. So, to minimise these errors, we repeat the experiments several times and adopt statistical techniques to get maximum precision.

The following are some other specific suggestions to minimise error.

1. Blanks experiments are to be conducted along with regular ones.
2. Atmost care is taken to avoid personal errors. Arithmetic mistakes, etc., should not be committed.
3. While taking readings one must be very careful to note the correct reading.
4. Without getting proper and complete instructions, the experiment should not be done.
5. When one becomes tired, the experiment must be stopped in a convenient place and continued after taking sufficient rest.

## 2. PRECISION

*Definition :*

It is the degree of aggrement between two or more measured values of a property measured under identical condition.

*Explanation with example :*

Let the weight of a beaker be 20.0326g when weighed ina particular set of conditions. If the same beaker is weighed under identical conditions and if the weight got is the same 20.0326g then we say that there is precision in the weighing. Thus if a value is reproducible then it called a precise value.

*Precision and range of a set of value :*

The differenses between the highest and lowest values in a set of values is called the range (w). This range is a measure of precision. If the range is wids then it means that the measurement is less precise.

## 3. ACCURACY

*Definition :*

It is the degree agreement between the measured value and the expected or true value of a property.

*Explanation with example :*

As per analytical data available the solubility product of AgCl is  $1.8 \times 10^{-10}$ . If an analyst gets this same value while determining the same

in an experiment, then. It is said accuracy of the result is excellent. If it differs, we calculate the deviation. This deviation gives a measure of the accuracy of experiment.

### Methods of expressing accuracy :

Accuracy is expressed in terms of absolute error or relative error. The lower these values are, the more will be the accuracy.

### Absolute Error (E) :

#### Definition :

It is defined as the difference between the accepted value ( $x_t$ ) and the observed values ( $x_i$ ). Mathematical expression :  $E = x_i - x_t$ .

### Relative Error (RE) :

#### Definition :

It is the error percentage the accepted value.

#### Mathematical expression :

$$RE = \frac{E}{x_t} \times 100$$

#### Advantage of relative error over absolute error :

Absolute error depends on the reliability of the accepted value itself as a lot of uncertainty may be there about the accepted value. So relative error is used more often to express accuracy.

### Distinctions between Absolute Error (E) and Relative Error (RE)

	E	RE
1. Definition	$E = x_i - x_t$	$RE = (E/x_i) \times 100$ $= (x_i - x_t / x_i) \times 100$
2. Usefulness	Not much useful as E depends on $x_t$ which itself is subject to uncertainty.	More useful



## Difference between Precision and Accuracy

Precision	Accuracy
1. Degree of agreement between a value and other values obtained under substantially the same condition.	Degree of agreement between a measured value and the true value or expected value
2. Precision may be achieved.	Accuracy is never known. It is known within certain limits only. Accuracy can be approached but never attained.
3. Expresses the reproducibility of the results.	Expresses correctness of a measurement.

One must clearly understand the connotations of the terms precision and accuracy. An understanding of the two terms will make it clear that high precision does not imply accuracy. We can prove this with an example. In a volumetric estimation one may get concordant titre values. Yet the result may turn out to be wrong. Here, as concordant titre values have been got, the result is precise.

But as the answer does not agree with the expected value, it is not accurate. Thus as precise value may not be an accurate value. The reason for this may be a determinate error like instrumental error or operative or personal error like personal carelessness or even some unknown indeterminate error.

### Confidence limits :

#### Definition :

They are the limits, which may be set, about the experimentally measured mean ( $\bar{x}$ ), within which we may expect to find the true mean ( $\mu$ ) with a given degree of probability.

**Explanation :**

Only the mean of an infinite number of measurements can be true or accurate mean. This true mean is represented by  $\mu$ . This is unattainable as an infinite number of measurements is impossible. In practice, we make a finite number of measurements and calculate the experimental mean ( $\bar{x}$ ). How near or how far away is  $\bar{x}$  from  $\mu$ ? This can be determined by setting limits. Within these limits we can find  $\mu$  with a given degree of probability. The interval between these limits is called the confidence interval. The size of this interval depends on the degree of probability that we need. If we want 99% probability, the interval will be small. 99% probability means that 99 times out of 100, the true mean will be within this interval. Similarly we can choose 95% probability, 90% probability etc., depending upon our needs. This percentage probability is called confidence level.

**Mathematical expression of confidence limits :**

a. When the standard deviation is for a small number of measurements,  $\sigma$  (standard deviation, applicable only when the number of measurements is large) the confidence limit for a single measurement for  $\mu$  is given by

$$\mu = \bar{x} \pm Z\sigma$$

$$\text{Where } Z = \frac{\bar{x} - \mu}{\sigma}$$

The confidence limit for the  $\mu$  of N measurements is given by

$$\mu = \bar{x} \pm \frac{Z\sigma}{\sqrt{N}}$$

When  $\sigma$  is unknown : Confidence limit for  $\mu$  of N measurements is given by

$$\mu = \bar{x} \pm \frac{ts}{\sqrt{N}}$$

Where 
$$t = \frac{\bar{x} - \mu}{s}$$

Thus we find by applying statistical methods we can fix the confidence limits within which the true average of a set of experimental results can be found for various confidence levels of probabilities. For this we must know the values of Z and t which are readily available in literature.

### Rejection of Results :

In a set of data we come across one or two values that are suspicious. Whether to reject such a data or not would become difficult. In such cases we employ a test called Q - test. In this test we compare two Q - values.

i.  $Q_{\text{exp}}$  and ii.  $Q_{\text{crit}}$  If  $Q_{\text{exp}} > Q_{\text{crit}}$  then we reject the data. If not, we retain the data.

To get  $Q_{\text{exp}}$  the difference between the suspicious value and its nearest neighbour is divided by the spread of the entire set.

To get  $Q_{\text{crit}}$  the following table giving various  $Q_{\text{crit}}$  values is used.

Number of observations	$Q_{\text{crit}}$		
	90% confidence	96% confidence	99% confidence
3	0.94	0.98	0.99
4	0.76	0.85	0.93
5	0.64	0.73	0.82
6	0.56	0.64	0.74
7	0.51	0.59	0.68
8	0.47	0.54	0.63
9	0.44	0.51	0.60
10	0.41	0.48	0.57

Let us illustrate how to decide whether to reject or retain a data.

Let us say that the experimentally determined percentages of CaO in a calcite sample are 55.95, 56.00, 56.04, 56.08, 56.23. Now we suspect that the last value and so we have to decide whether to reject it or not.

$$\begin{aligned} \text{For this set of data } Q_{\text{exp}} &= (\text{Suspectious value} - \text{Its nearest neighbour}) \\ &\div \text{the spread of the entire set.} \\ &= (56.23 - 56.08) \div (56.23 - 55.95) \\ &= 0.15 \div 0.28 = 0.54 \end{aligned}$$

Now let us say we want our results in 90% confidence level i.e., our result should be 90% as near to the correct value then we look under 90% confidence column of the table. Since we have 5 observations we look for  $Q_{\text{crit}}$  against number of observations 5 and under 90% confidence. The value is 0.64.

Since  $Q_{\text{exp}} < Q_{\text{crit}}$  we conclude that the value is to be retained.

Though the Q - test is superior to other methods one must be careful while deciding to either reject or retain a particular data using this test. One has to use one's good judgement in deciding. Where the number of observations are small the reliability of Q - test decreases. So, to reject a value in a small set a cautious approach is desirable.

#### 4. SIGNIFICANT FIGURES

They are figures in a number which contains only digits known with certainty plus the first uncertain one.

##### *Explanation :*

A measured value has some uncertainty about it. There is a convention to give the measured value as a number such that it contains only one figure about which there is uncertainty. The practice is called significant figure convention.

##### *Example :*

If the weight in a weighing is known with certainty only upto three decimals, the value should be reported only upto four decimals.

Salient features ? Points to be borns in mind while using the concept of significant figures :

The number of significant figures in a given number is found out by counting the number of figures from left to right in the number beginning with the first non - zero digits and continuing till reaching the digit that contains the uncertainty.

### *Examples :*

1. Each of the following has three significant figures. 58.2, 0.234, 1.67, 0.00987 and 65.4.
2. Zero is a significant figure when used as a number. It is not a significant figure when it is used to locate decimal points in very small and very large numbers. E.g., 0.02670 has four significant figures. The two zeros before 2 are used to imply only the magnitude. So they are not significant. The zero beyond 7 is significant.
3. The value  $6.030 \times 10^{-1}$  has four significant figures while  $1.45 \times 10^5$  has three significant figures.

Using the above, one can determine the number of significant figures in a given number.

### *Its importance or Uses :*

In presenting scientific data, one comes across a set of values. For this set of values, one gives their mean or median as the best value. Now the uncertainty about this best value must also be indicated while presenting the data. To achieve this, usage of significant figures is very helpful. On several occasions one has to round off numbers to give meaningful results. For this, the practise of rounding off to a number which contains only one uncertain figure in it, is adopted. Thus significant figures become important.

### **Methods of expressing Precision :**

Precision is expressed by two methods.

- i. Absolute Method
- ii. Relative Method.

**i. Absoulte Method :**

In this method the precision is expressed in terms of average mean deviation. The smaller the value of average mean deviation the greater will be the precision.

**ii. Relative Method :**

In this method the precision is expressed in terms of percentage deviation from the mean of a set of values.

$$\left. \begin{array}{l} \text{i.e., Percentage deviation} \\ \text{from the mean} \end{array} \right\} = \frac{\begin{array}{l} \text{Mean of a set of values} \\ \text{minus a particular value} \end{array}}{\text{The mean}} \times 100$$

$$= \frac{\bar{x} - x_i}{\bar{x}} \times 100$$

To understand the above two methods let us learn some terms used in statistics.

**Mean / Arithemetic mean / Average :**

Mean is the quotient obtained when the sum of a set of replicate measurements by the number of individual results in the set. The following worked out unversity problem would illustrate mean.

**Problem :**

Calculate the mean for the following set of values 20.21, 20.04, 20.13 and 20.19

**Solution :**

$$\text{Mean} = (20.21 + 20.04 + 20.13 + 20.19) \div 4 = 80.57 \div 4 = 20.14.$$

**Median :**

It is the value about which all other values are equally distributed. Half the values will be greater and the other half smaller than the median.

The median is obtained by arranging the set of values in increasing or decreasing order. If there are odd number of values then the middle

value gives the median. If there are even number of values than the average of the middle pair gives the median. The following worked out university problems would illustrate the median.

**Problem :**

Calculate the median for 20.20, 20.08 and 20.02.

**Solution :**

Arranging the values in increasing order we get 20.01, 20.08 and 20.20. This set contains odd number of values. Therefore the median of this set is the middle value i.e., 20.08.

**Problem :**

Calculate the median for the following set of values 20.21, 20.04, 20.13 and 20.19.

**Solution :**

Arranging the values in increasing order we get 20.04, 20.13, 20.19 and 20.21. This set contains even number of values. Therefore the median of this set is the average of the middle pair of values, i.e.,  $(20.13 + 20.19) \div 2 = 40.32 \div 2 = 20.16$

### Difference between Mean and Median

	Mean	Median
1. Definition	It is the quotient obtained when the sum of a set of replicate measurements by the number of individual results in the set.	It is the value about which all other values are equally distributed 20.08
2. Examples	20.09 Date : 20.20 20.08, 20.01	20.08
3. Precision	Better than median	Less than mean

# Mean deviation or Average deviation

## Definition :

The average deviation of a value in a set of values is the average of the deviations of all the individual values from their average.

## Explanation :

To get the average deviation.

- i. The average of the given set of values is calculated.
- ii. The deviation of each value from the average is calculated.
- iii. The average of all these deviations (ignoring signs) give the average deviation. The following worked out University problems would illustrate average deviation.

## Problem :

Calculate the mean (average) deviation for the following set of values : 20.21, 20.04, 20.13 and 20.19.

## Solution :

$$\begin{aligned} \text{Average of this set} &= (20.21 + 20.04 + 20.13 + 20.19) \div 4 \\ &= 80.57 \div 4 = 20.14. \end{aligned}$$

Value	Deviation from average
20.21	$20.14 - 20.21 = 0.07$
20.04	$20.14 - 20.04 = 0.10$
20.13	$20.14 - 20.13 = 0.01$
20.19	$20.14 - 20.19 = 0.05$
<hr/>	
Total of deviation from average	$= 0.07 + 0.10 + 0.01 + 0.05$ $= 0.23$
Mean deviation	$= 0.23 \div 4 = 0.575$

## Its usefulness :

If the mean deviation of a set of measurements is small it means that the average of that set is nearly precise.



## Standard Deviation :

### Definition :

It is the square root of the quotient obtained by dividing the sum of the squares of the individual deviations from their mean by the number of measurements made.

or

$$\text{Standard deviations} = \sqrt{\frac{\text{Sum of squares of individual deviation from their mean} \div \text{Number of measurements made}}{}}$$

### Explanation :

To obtain standard deviation.

- i. The average ( $\bar{x}$ ) of the measurements ( $x_{ii}$ ) is calculated.
- ii. The individual deviation of each measurement from the average ( $x_i - \bar{x}$ ) is calculated.
- iii. Each individual deviations is squared ( $(x_i - \bar{x})^2$ )
- iv. All the individual deviation square are added  $\sqrt{\Sigma(x_i - \bar{x})^2}$ .
- v. The value obtained in step.
- vi. It is divided by the number of measurements made  $\Sigma(x_i - \bar{x})^2 \div N$ .
- vii. The square root of the value obtained in step.
- v. Gives the standard deviation. Thus standard deviation  $\sigma = \sqrt{\Sigma(x_i - \bar{x})^2 \div N}$ .

$\sigma$  is applicable only when the number of measurements is large. But in analytical chemistry we make only a small number of measurements. so in step (v) instead of dividing by N the value obtained in step (iv) is divided by  $N - 1$ . The standard deviation for a small number of measurements  $s = \sqrt{\Sigma(x_i - \bar{x})^2 \div (N - 1)}$ .

### Example / Illustration :

The following worked out university problem would illustrate how standard deviations is obtained from a set of data.

### Problem :

Find the standard deviation for a subset having the following six values.

7.720, 7.725, 7.736, 7.719, 7.742 and 7.751

**Solution :**

i. Calculation of average  $\bar{x}$

$$(7.720 + 7.725 + 7.736 + 7.719 + 7.742 + 7.751) \div 6$$

$$= 46.393 \div 6 = 7.732$$

ii. Calculation of  $\Sigma(x_i - \bar{x})^2$

$x_i$	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
7.720	0.012	$1.44 \times 10^{-4}$
7.725	0.007	$0.49 \times 10^{-4}$
7.736	0.004	$0.16 \times 10^{-4}$
7.719	0.013	$1.69 \times 10^{-4}$
7.742	0.010	$1.00 \times 10^{-4}$
7.775	0.043	$18.49 \times 10^{-4}$

iii.  $\therefore \Sigma(x_i - \bar{x})^2 = 23.37 \times 10^{-4}$

iv.  $\frac{\Sigma(x_i - \bar{x})^2}{N - 1} = \frac{23.27 \times 10^{-4}}{5} = \frac{4.82}{5} = 0.96$

v.  $\sqrt{\Sigma(x_i - \bar{x})^2} \div (N - 1) = 0.98$

Standard deviation = 0.98.

### Exercises : (Univeristy Problems)

- Calculate the standard deviation for the subset of the values : 25.32, 25.07, 25.18, 25.26  
[Ans : 0.1082]
- The percentage of a constituent M in a compound MA are found to be 62.42, 62.28, 62.46, 62.32 and 62.22. Calculate the mean, median and standard deviation.  
[Ans : i. 62.34, ii. 62.32, iii. 0.099]

Advantages of standard deviation : Standard deviation is more reliable than average or mean deviation to express precision as it has theoretical foundation.

### Its usefulness :

If the standard deviation of a set of measurements is small, it means that the average of the set is nearly precise.

## Difference between Mean deviation and Standard deviation

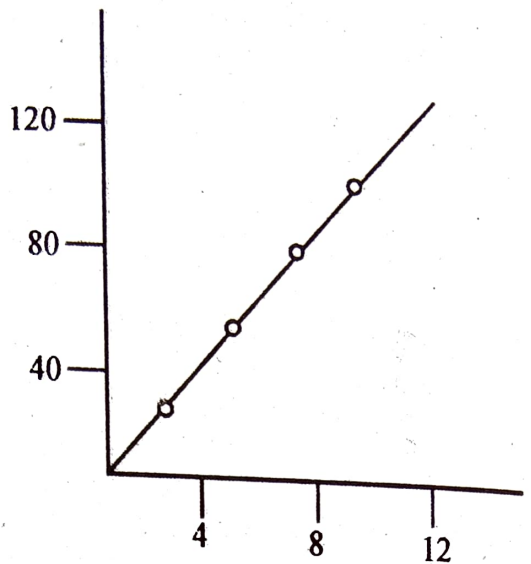
	Mean deviation	Standard deviation
1. Definition	It is the average of the deviation of all the individual values from their average.	Sum of squares of individual deviation from their mean ÷ number of measurements made.
2. Formula	$\sqrt{\Sigma(x_i - \bar{x}) \div N}$	$\sqrt{\Sigma(x_i - \bar{x})^2 \div (N - 1)}$
3. Precision	Less than standard deviation but better than average	Better than mean deviation, It is the best way of expressing precision.

## Curve Fitting - Method of Least Squares

If we want to present a trend or relationship we draw a graph. A graph is obtained by plotting two variables X and Y against each other. Let us consider the following data.

x	y
2	20.20
4	40.4
6	60.6
8	80.2
10	100.8
12	120.0

A graph may be drawn using these data (figure). By convention we plot the independent variable, i.e., the cause of the value based on which a particular prediction is made along the axis (horizontal axis) and the dependent variable. i.e., the effect on the predicted property, along the Y axis (vertical axis).



The graph gives the trend or relationship between X and Y. The above graph indicates a linear relationship between X and Y. In the above graph linearity is good. Usually for the data obtained in experiments, linearity will not be as good as shown in figure. In such cases we have to draw a best fit line. This is called curve fitting. For this purpose we use the method of least squares. By this method we get a straight line for which the algebraic sum of the vertical deviations is less than that from any other straight line. There will be only one such line i.e., it will be the best fit line. This best fit line cannot be drawn just by observing the points on the graph. For drawing the best fit line we have to use the method of least squares.

Let us consider the following data :

X	Y
1	1.5
2	1.8
3	2.7
4	4.0

To draw the best fit line for the above data the following steps are followed.

- i.  $\Sigma X$ ,  $\Sigma Y$ ,  $\Sigma XY$  and  $\Sigma X^2$  are calculated.

In our example,

$$\Sigma X = 10, \Sigma Y = 10, \Sigma XY = 29.2 \text{ and } \Sigma X^2 = 30$$

- ii. The above values are substituted in the following simultaneous equations.

$$\Sigma Y = aN + b\Sigma X$$

$$\Sigma XY = a\Sigma X + b\Sigma X^2$$

Here  $N$  = the number of pairs of data = 4

$$\text{i.e., } 10 = a \times 4 + b \times 10 \quad - \quad (1)$$

$$\text{and } 29.2 = a \times 10 + b \times 30 \quad - \quad (2)$$

multiplying (1) by 3 we have

$$30 = 12a + 30b \quad - \quad (3)$$

subtracting (2) from the (3) we have

$$0.8 = 2a$$

$$\therefore a = 0.4 \quad - \quad (4)$$

Substituting (4) in (1) we have  $b = 0.84$

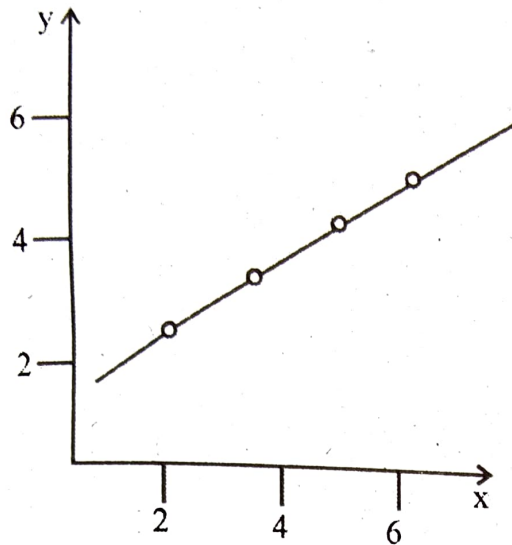
- iii. Now for the given values of  $X$ ,  $Y$  values are calculated using the equation

$$Y_{\text{cal}} = a + bX$$

The calculated  $Y$  values for two values of  $X$  from our data are calculated.

$X$	$Y_{\text{cal}} = (0.4 + 0.8)X$
1	1.2
4	3.6

iv. Two points corresponding to these two pairs of values are marked on the graph and they are joined. We get the best fit line for our data. The following will be the graph for the data given.



### Correlation Coefficient :

It is a quantity that indicated the extend of linearity of a given set of data. It is denoted by  $r$ .

#### *Mathematical expression.*

$$r = \frac{N\sum XY - (\sum X)(\sum Y)}{\{[N\sum X^2 - (\sum X)^2] [N\sum Y^2 - (\sum Y)^2]\}^{1/2}}$$

Where  $X$  and  $Y$  are variables or two properties,  $N$  is the number of pairs of data.

#### *Explanation :*

It helps us find out whether there is any linear relationship between two properties  $X$  and  $Y$  from  $N$  pairs of data relating  $X$  and  $Y$ .

If  $r$  is 1.0 it means that there is perfect linearity relationship between  $X$  and  $Y$ .

If  $r$  is between 0.99 and 0.75 it means that there is excellent linearity relationship between  $X$  and  $Y$ .

If  $r$  is between 0.75 to 0.50. It means that there is good linearity relationship between  $X$  and  $Y$ .

If it is less than 0.50 we can conclude that there is poor linearity relationship between X and Y.

In our example given above

$$\begin{array}{lll}
 N=4 & \Sigma X = 10 & \Sigma Y = 10 \\
 \Sigma XY = 29.2 & \Sigma X^2 = 30 & (\Sigma X)^2 = 100 \\
 \Sigma Y^2 = 28.78 & (\Sigma Y)^2 = 100 & 
 \end{array}$$

$$\begin{aligned}
 r &= \frac{(4 \times 29.2) - (10 \times 10)}{\{[4 \times 30 - 100][4 \times 28.78 - 100]\}^{\frac{1}{2}}} \\
 &= \frac{116.8 - 100}{\{[120 - 100][115.12 - 100]\}^{\frac{1}{2}}} \\
 &= \frac{16.8}{\{[20][15.12]\}^{\frac{1}{2}}} = \frac{16.8}{304^{\frac{1}{2}}} \\
 &= \frac{16.8}{17.44} = 0.96
 \end{aligned}$$

That is, as per our data, there is excellent linear relationship between X and Y.

Thus correlation co-efficient gives us an idea about the extent of linearity of a given set of data.

## Chemical and single pan balance



### Precautions in using an analytical balance

1. Center the load on the pan as well as possible.
2. Protect the balance from corrosion.
3. Consult the instructor if the balance appears to need adjustment.
4. Keep the balance and its case scrupulously clean. A camel-hair brush is useful for removing spilled materials or dust.

5. Always allow an object that has been heated to return to room temperature before weighing it.
6. Use tongs of finger pads to prevent the uptake of moisture a by dried object.

### Buoyancy effect

A buoyancy error is the weight error that develops when the object being weighed has a significantly different density than the masses. The buoyancy error will affect data if the density of the object being weighed differs significantly from that of the standard masses. This error has its origin in the difference in buoyant force exerted by the medium (air) on the object and on the masses. Buoyancy corrections for electronic balance may be accomplished use the equation.

$$W = W_1 + W_2 \left( \frac{d_{\text{air}}}{d_{\text{obj}}} - \frac{d_{\text{air}}}{d_{\text{wts}}} \right)$$

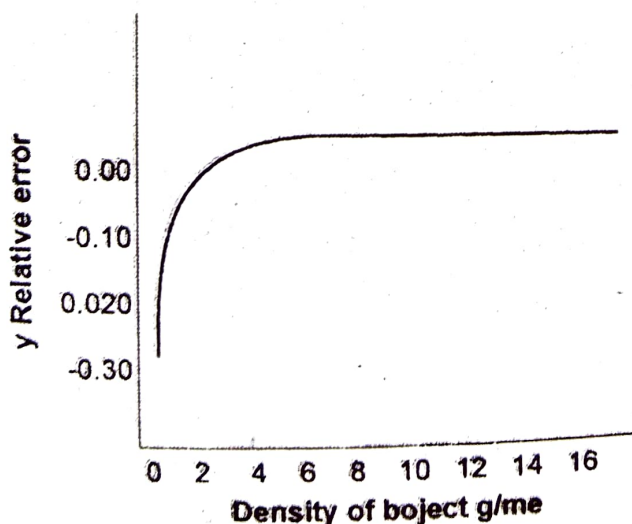
$W_1$  - Corrected mass of the object

$W_2$  - Mass of standard mass

$d_{\text{obj}}$  - Density of the object

$d_{\text{air}}$  - Density of air (0.00129/cm<sup>3</sup>)

$d_{\text{wts}}$  - Density of masses

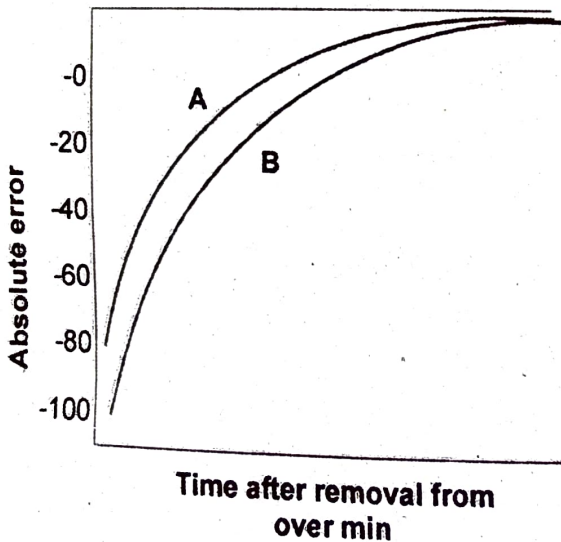




The consequence of equation are shown in figure in which the relative error due to buoyancy is plotted against the density of object weighted in air against stainless steel masses. Note that this error is less than 0.1% for objects that have a density of  $2\text{g/cm}^3$ .

The density of masses used in single pan balance ranges from  $7.8$  to  $8.4\text{g/cm}^3$  depending on the manufacture. Use  $8\text{g/cm}^3$  is adequate for most purpose is greater accuracy is required the specifications for the balance to be used should be consulted for the necessary density data.

### Temperature effects



Affects to weight an object whose temperature is difference from that of its surroundings will result in a significant errors. Errors due to a difference in temperature have two sources. First convection current within the balance case exert a buoyant effect on the pan and object. Second warm air trapped in a closed container weights less than the same volume at low temperature both effects cause the apparent mass of the object to be low. The error can amount to as much of 10 (or) 15 mg for typical porcelain filtering crucibles (or) weighting bottle.