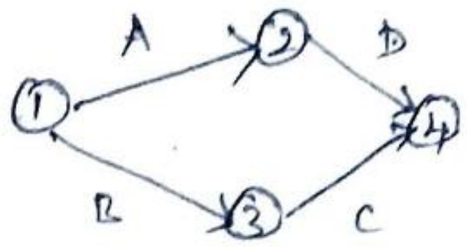
# Unit V : Network.

1. Network :

A network is a graphical representation of project operation which is logically and sequencially connected by arrows and nodes.



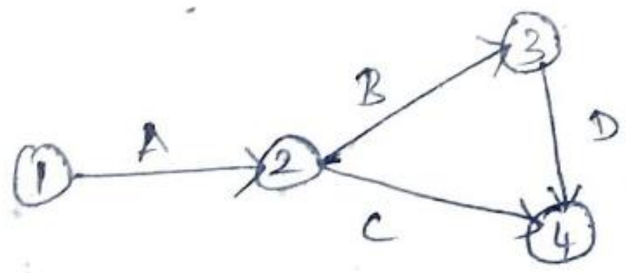

2. Activity :

The entire task of the project can be sub divided into smaller elements. These elements of the project is called an activity. Activities are represented by a line with an arrow showing the direction.

3. Types of Activities :

Predecessor Activity : [Preceding Activity]

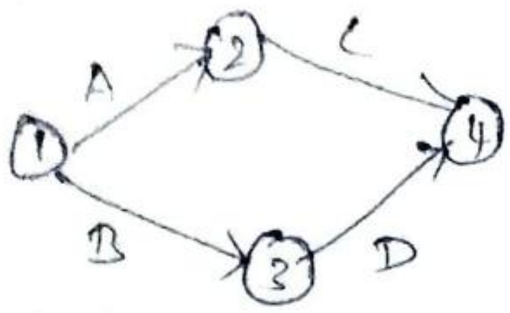
Activity which must be completed before a particular activities start is called the predecessor activity.




Here A is the predecessor activity of B & C.

Successor Activity [Succeeding Activity]

Activity which cannot be started one or more of other activities are completed but immediately succeed them is called successor activity.

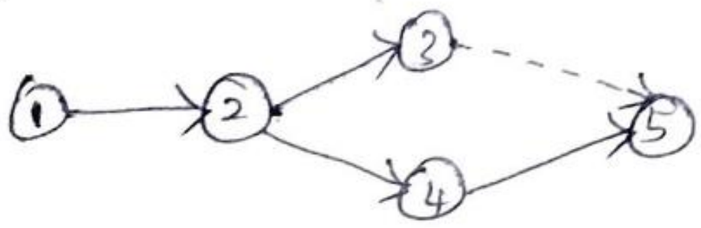



Here C is the successor activity of A

D is the " " of B.

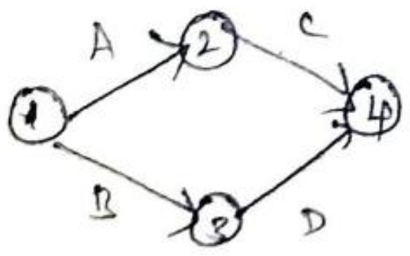
## Dummy Activity:

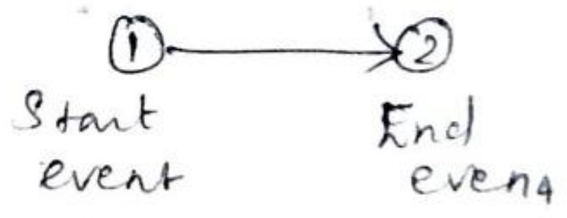
An activity which does not consume any kind of resource but merely explains the technological depend is called a dummy activity. It is represented by dotted arrow lines.




4. Event:

An event is also called a state or node. indicates the start or end of the jobs. It is represented by the circle and the event number is written with in the circle.

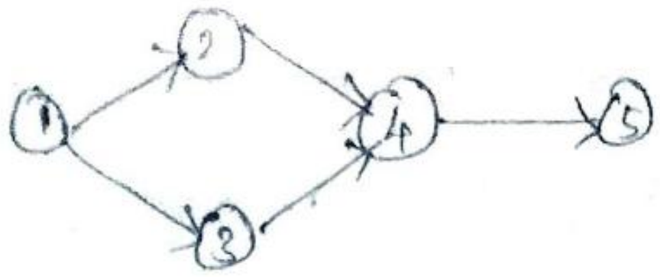



5. Types of Event:

## Merge Event:

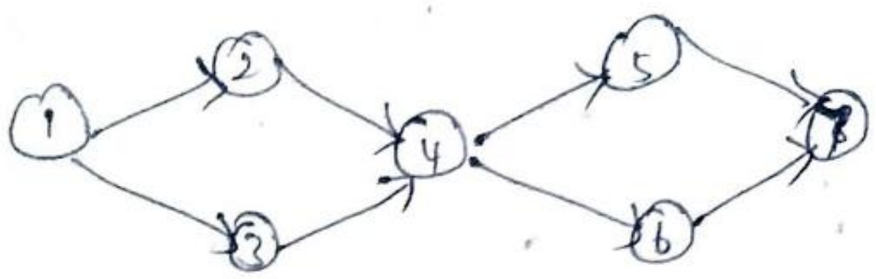
When more than one activity comes and joints the event in known as merge event




Here, event (4) is called merge event

Burst Event :

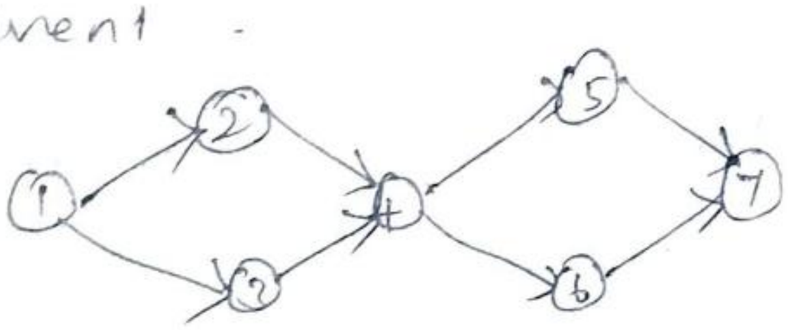
When more than one activity leaves is called a Burst event.




Here, event (1) and (4) are called Burst event.

Merge and Burst Event :

An activity which is a merge and burst event simultaneously then it is called merge and Burst event.




Here, event (4) is called merge and burst event.

6. Fulkerson's rules of drawing a network diagram :

(i) Each activity is represented by only one arrow

(ii) Time flows from left to right.

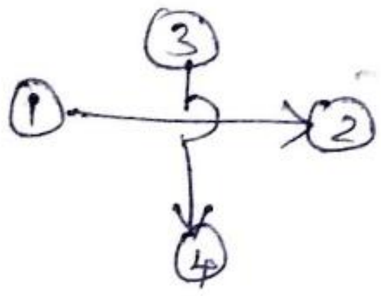
(iii) Arrows pointing in opposite direction are to be avoided.

(iv) Events should be numbered such that for activity (i,j), where i < j.

(V). The event numbered (1) denotes the start of the project and it is called initial node. The event carrying the highest number denotes the Completion of an event.

(Vi) Arrows should be kept straight and not curved or bend.

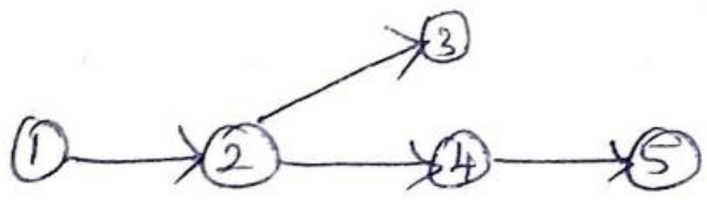
(Vii) Arrows should not cross each other, where Crossing can not be avoided, the crossing methods shown in the figure can be adopted.




7 Common Errors in drawing a network!

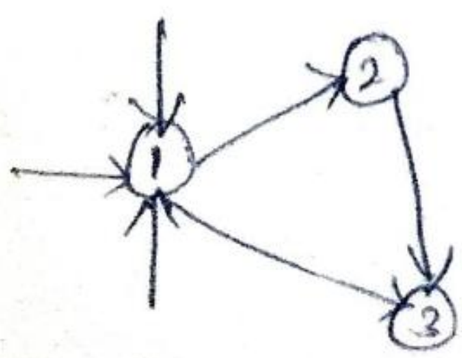
Dangling:

Disconnecting an activity before the Completion of all the activities in a network diagram is called as Dangling.



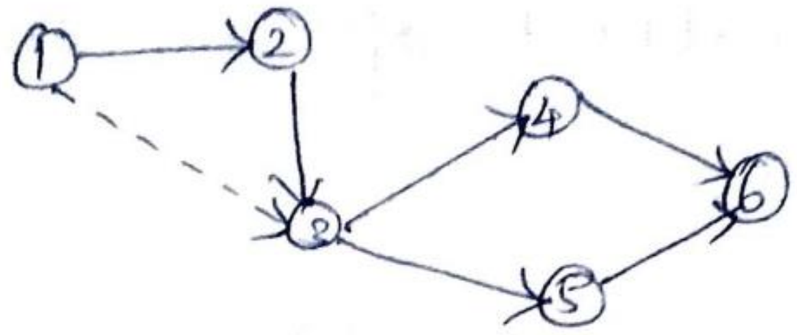

Looping:

Looping Error is also called as cycling error. Drawing an endless looping in a network is known as looping error.



Redundaney :

Providing unnecessary dummy activities in a network diagram is known as Redundaney.




8. Numbering the events :

(i). Event numbers should be unique.

(ii) Event numbering should be carried out on a sequential basis from left to right.

(iii) The initial event which has all out going arcows with no incoming arrow is numbered

0 (or) 1

(iv). The head of an arrow should always bear a number higher than the one assigned at the tail of the arrow.

(v) Gaps should be left in the sequence of event numbering to accomodate subsequent inclusion of activities, if necessary.

9. Critical activity :

An activity in a network diagram is said to be critical, if the delay in its start will further delay the project Completion time.

10. Non-Critical activity:

An non-critical activity allows some scheduling slack, so that the start time of the activity may be advanced or delayed with in limits without affecting the completion date of the entire project.

11. Critical pathmethod (or) CPM.:

The iterative procedure of determining the critical path is as follows:

Step:1

List all the jobs and then draw a network diagram. Each job is indicated by an arrow with the direction of the arrow showing the sequence of jobs. The length of the arrows has no significance. Then place the jobs on the diagram one by one keeping in mind what proceeds and follows each job as well as what job be done simultaneously.

Step:2

Consider the jobs times to be deterministic. Indicate them above the arrow representing.

Step: 3

calculate the earliest start time (EST) and earliest finish time (EFT) for each event and write them in the box marked [✓|  ]. Calculate the latest start time (LST) and latest finish time (LFT) and write them in the box marked

[  |✓]

Step: 4

Tabulate various time. (i.e) activity normal times, earliest times and latest times and mark EST and LFT on the arrow diagram.

Step: 5

Determine the total float for each activity by taking difference between EST and LFT

Step: 6

Identify the critical activities and connect them with the beginning node and the ending node in the network diagram by double line arrow. This gives the critical path.

Step: 7

Calculate the total project duration.

12 PERT: Algorithm:

The various steps involved in developing PERT network for analysing any project are summarized below:

Step: 1

Make a list of activities that make up the project including immediate predecessors.

Step: 2

Make use of step 1, sketch the required network.

Step: 3

Denote the most likely time by $t_m$, the optimistic time by $t_o$, and pessimistic time by $t_p$.

Step: 4

Using beta distribution for the activity duration, the expected time $t_e$, for each activity is computed by using the formula:

$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

Step: 5

Tabulate various times.

i.e. expected activity times, earliest and latest times and mark the EST and LFT on the

arrow diagram.

step: 6

Determine the total float for each activity by taking the difference between EST and LFT.

step: 7

Identify the critical activities and connect them with the beginning node and the ending node, in the network diagram by double line arrows. This gives the critical path and the expected date of completion of the project.

step: 8

Using the values of tp and to, compute the variance ($\sigma^2$) of the each activity's time estimates by using the formula

$$\sigma^2 = \left(\frac{tp - to}{6}\right)^2$$

step: 9

Compute the standard normal deviate.

$$Z_o = \frac{\text{Due date} - \text{Expected date of completion}}{\sqrt{\text{Project variance}}}$$

step: 10

Use standard normal tables to find the probability $P(Z \leq Z_o)$ of completing the project within the scheduled time, where $Z \sim N(0,1)$.

13. Three time Estimates used in PERT :

Optimistic Time :

The optimistic is the shortest possible time estimate for finishing an activity. The chance of occurance of this small. This time is denoted by $t_o$ (or) a.

Most Likely Time :

This is the time estimate to be executed under normal conditions of the activity, This is a reasonable time. This is denoted by $t_m$ (or) m.

Pessimistic Time :

The pessimistic time is the largest time estimate for finishing an activity. The Chance of occurance of this is also small, it is denoted by $t_p$ (or) b.

14. Difference between PERT and CPM.

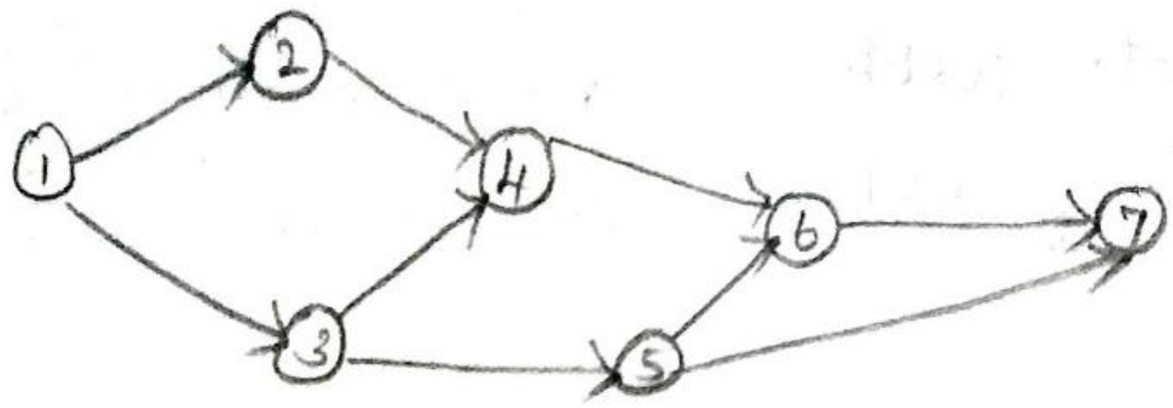
| CPM. | PERT |
|---|---|
| (i) CPM is used in construction projects. | PERT technique is applied to research and development projects. |
| (ii) CPM is used for repetitive and non complex projects. | PERT is used for non-repetition and complex projects |

| | |
|---|---|
| (iii) CPM is deterministic model | PERT is probabilistic model |
| (iv) The time estimate can be made with measure of certainity | The time estimates are uncertainity. |
| (v). CPM deals with the minimum of cost of projects | PERT analysis does not consider cost. |
| (vi) CPM includes time cost trade offs | PERT gives its attention to time variable. |
| (vii) CPM does not incorporate any statistical analysis. | PERT incorporate statistical analysis |
| (viii) The network concentrates more on activities in CPM. (ie) CPM is activity Oriented. | The network Concentrates more on events in PERT. (ie) PERT is event oriented. |

## Draw the network

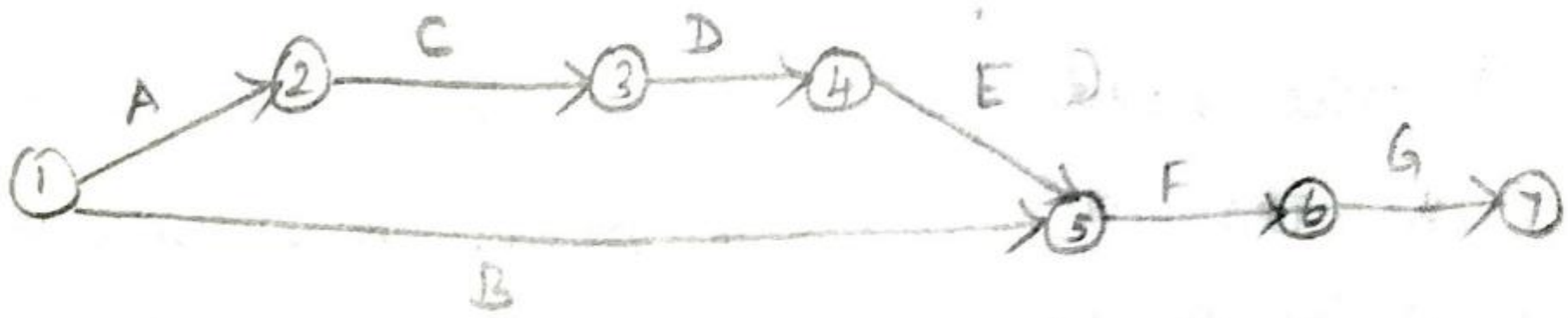
1) Event no : 1 2 3 4 5 6 7

Preceded by : start 1 1 2,3 3 4,5 5,6



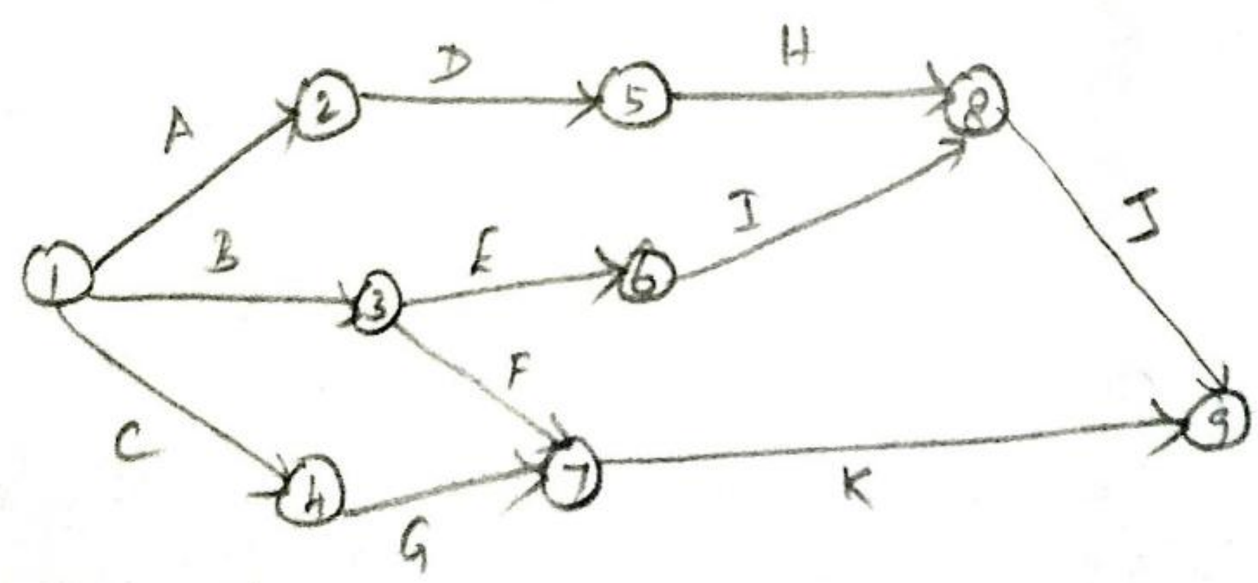

2. Activity : A B C D E F G.

Immediate Predecessor : — — A C D B,E E,F.

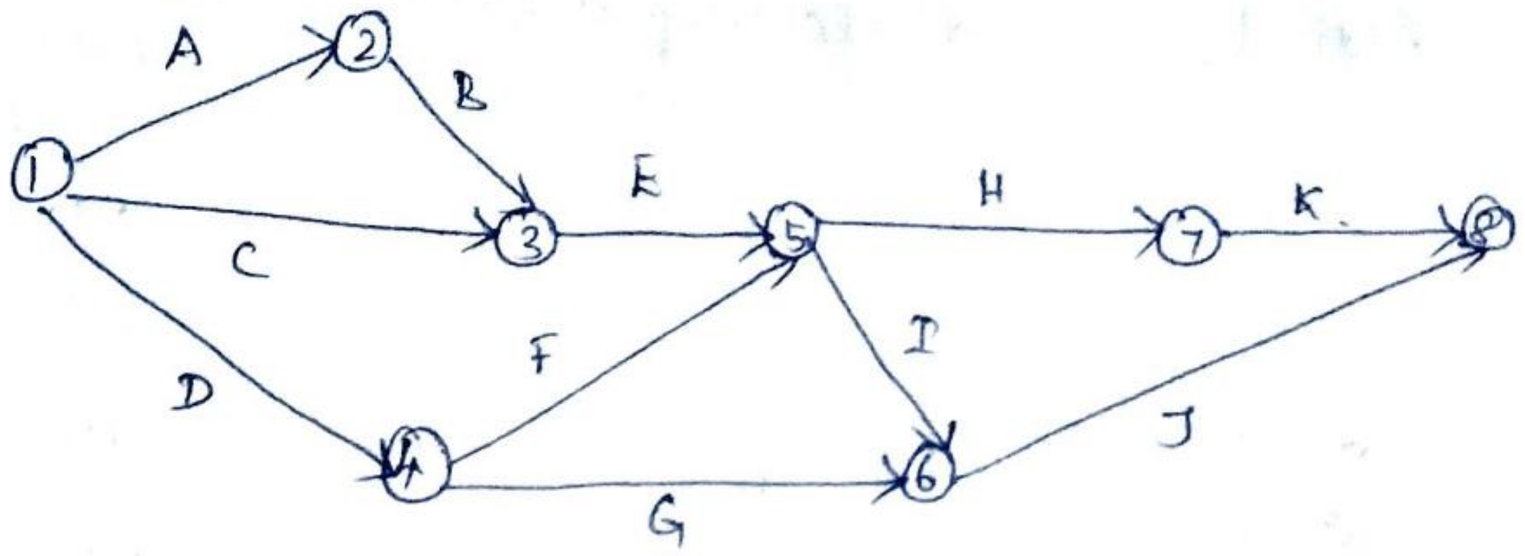



3. Activity : A B C D E F G H I J K

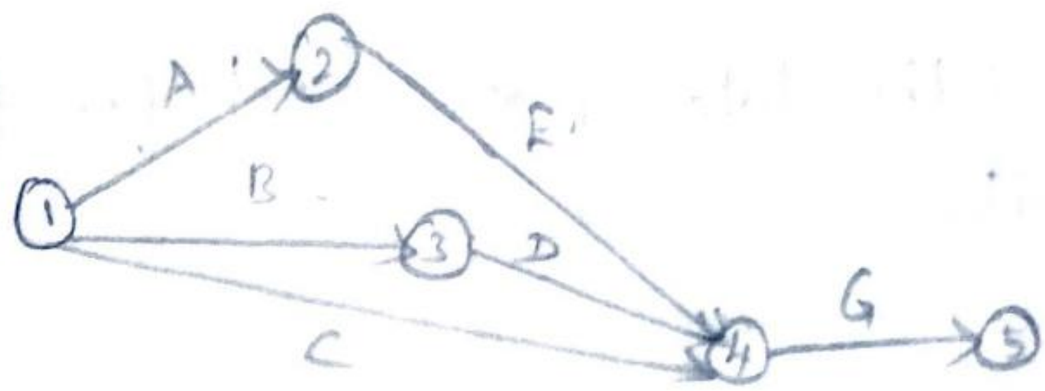
Immediate Predecessor : — — — A B B C D E H,I F,G.



4. $E > B, C$ ; $F, G > D$ ; $H, I > E, F$ ; $J > I, G$ ; $K > H$ ; $B > A$.

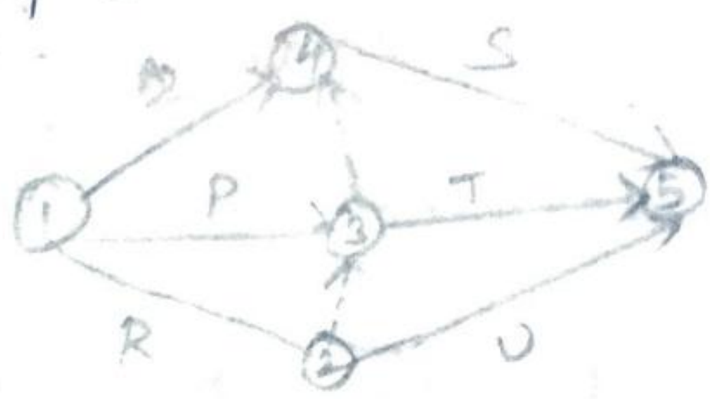



5. $A < E, F$ ; $B < D$ ; $C, E, D < G$

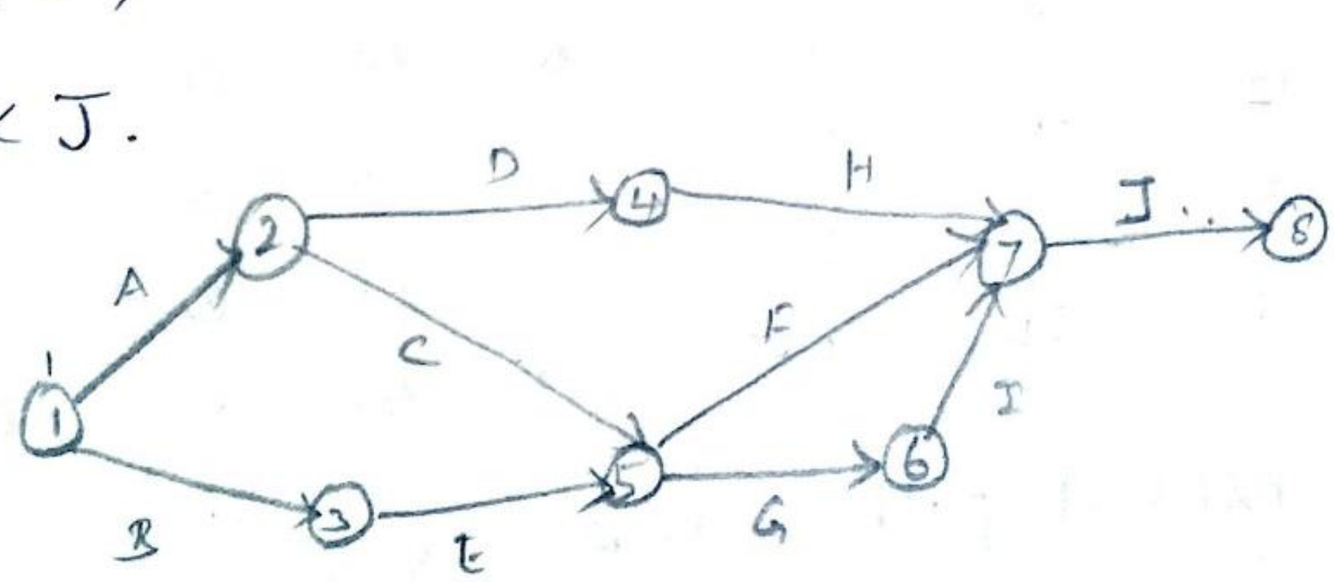



6. Activity : P Q R S T U

Preceding Activity : — — — P, Q P, R R.




7. $A < C, D$ ; $B < E$ ; $C, E < F, G$ ; $D < H$ ; $G < I$ ; $H, I < J$.



Critical Path Method

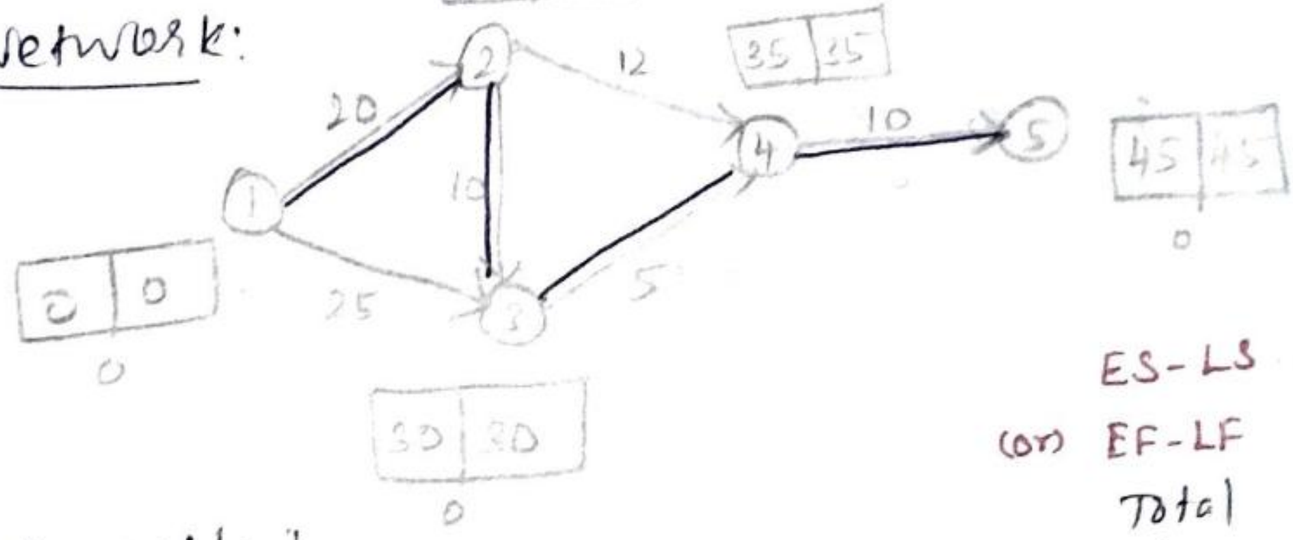
1. Find the critical path and project duration of the following: 2

| Activity | Preceding activity | Normal Time (days) |
|---|---|---|
| 1-2 | — | 20 |
| 1-3 | — | 25 |
| 2-3 | 1-2 | 10 |
| 2-4 | 1-2 | 12 |
| 3-4 | 1-3, 2-3 | 5 |
| 4-5 | 2-4, 3-4 | 10. |

Also, find the total float and freefloat for each activity

Soln:

Network:




Float table:

| Act | Time | EST | EFT | LST | LFT | ES-LS or EF-LF Total Float | Total Float - Head event slack Free Float |
|---|---|---|---|---|---|---|---|
| (1-2) | 20 | 0 | 20 | 0 | 20 | 0 ✓ | 0 |
| 1-3 | 25 | 0 | 25 | 5 | 30 | 5 | 5 |
| (2-3) | 10 | 20 | 30 | 20 | 30 | 0 ✓ | 0 |
| 2-4 | 12 | 20 | 32 | 23 | 35 | 3 | 3 |
| (3-4) | 5 | 30 | 35 | 30 | 35 | 0 ✓ | 0 |
| (4-5) | 10. | 35 | 45 | 35 | 45 | 0 ✓ | 0 |

Critical path = 1-2-3-4-5

Project Duration = 45 days.

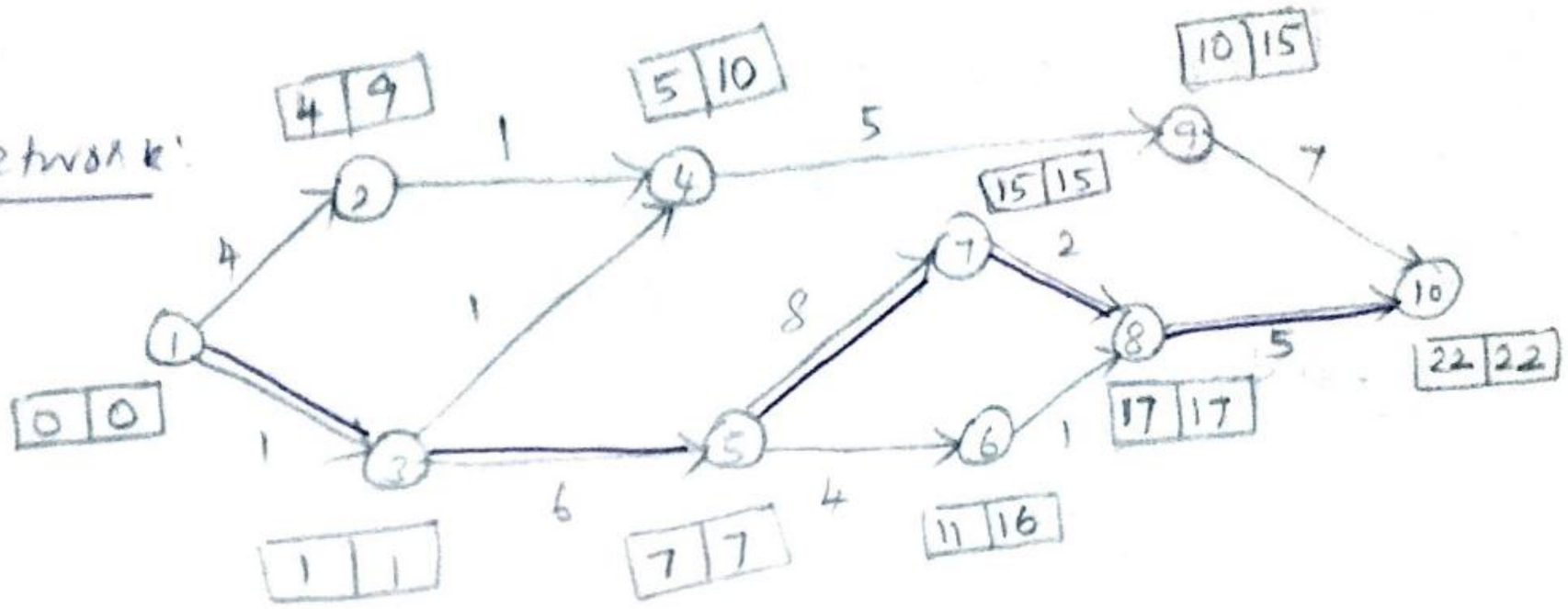
2. Find the critical path.

| Activity | 1-2 | 1-3 | 2-4 | 3-4 | 3-5 | 4-9 | 5-6 | 5-7 | 6-8 |
|---|---|---|---|---|---|---|---|---|---|
| time | 4 | 1 | 1 | 1 | 6 | 5 | 4 | 8 | 1 |

| 7-8 | 8-10 | 9-10. |
|---|---|---|
| 2 | 5 | 7. |

Soln:

Network:




Float Table:

| Activity | Time | EST | EFT | LST | LFT | Float |
|---|---|---|---|---|---|---|
| 1-2 | 4 | 0 | 4 | 5 | 9 | 5 |
| (1-3) | 1 | 0 | 1 | 0 | 1 | 0 ✓ |
| 2-4 | 1 | 4 | 5 | 9 | 10 | 5 |
| 3-4 | 1 | 1 | 2 | 9 | 10 | 8 |
| (3-5) | 6 | 1 | 7 | 1 | 7 | 0 ✓ |
| 4-9 | 5 | 5 | 10 | 10 | 15 | 5 |
| 5-6 | 4 | 7 | 11 | 12 | 16 | 5 |
| (5-7) | 8 | 7 | 15 | 7 | 15 | 0 ✓ |
| 6-8 | 1 | 11 | 12 | 16 | 17 | 5 |
| (7-8) | 2 | 15 | 17 | 15 | 17 | 0 ✓ |
| (8-10) | 5 | 17 | 22 | 17 | 22 | 0 ✓ |
| 9-10. | 7 | 10. | | 15 | 22 | 5 |

Critical path = 1-3-5-7-8-10.

Project duration = 22 days.

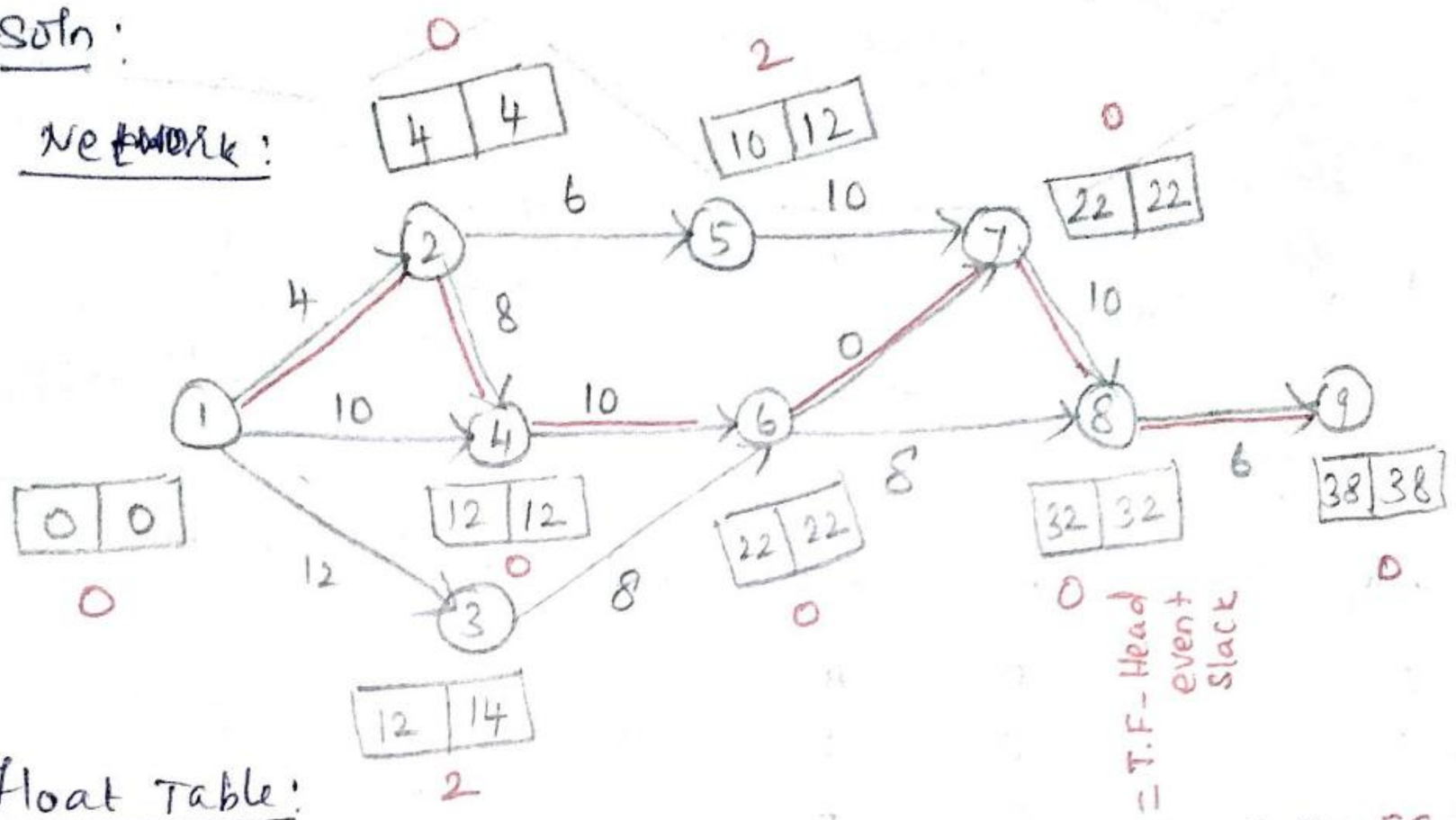
3. Find the critical path, Total float, Free float and independent float.

| Activity | 1-2 | 1-3 | 1-4 | 2-4 | 2-5 | 3-6 | 4-6 |
|---|---|---|---|---|---|---|---|
| Duration (Hrs) | 4 | 12 | 10 | 8 | 6 | 8 | 10 |

| 5-7 | 6-7 | 6-8 | 7-8 | 8-9 |
|---|---|---|---|---|
| 10 | 0 | 8 | 10 | 6 |

Soln:

Network:



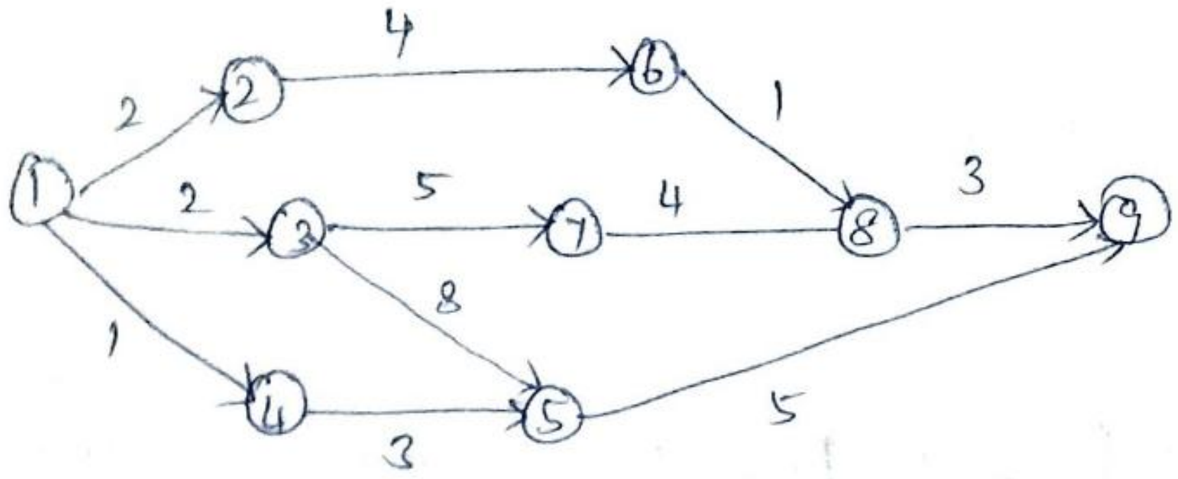

Float Table:

| Act | Time | EST | EFT | LST | LFT | TF | FF | IF = FF − Tail event slack |
|---|---|---|---|---|---|---|---|---|
| 1-2 | 4 | 0 | 4 | 0 | 4 | 0 ✓ | 0 | 0 |
| 1-3 | 12 | 0 | 12 | 2 | 14 | 2 | 0 | 0 |
| 1-4 | 10 | 0 | 10 | 2 | 12 | 2 | 2 | 2 |
| 2-4 | 8 | 4 | 12 | 4 | 12 | 0 ✓ | 0 | 0 |
| 2-5 | 6 | 4 | 10 | 6 | 12 | 2 | 0 | 0 |
| 3-6 | 8 | 12 | 20 | 14 | 22 | 2 | 2 | 0 |
| 4-6 | 10 | 12 | 22 | 12 | 22 | 0 ✓ | 0 | 0 |
| 5-7 | 10 | 10 | 20 | 12 | 22 | 2 | 2 | 0 |
| 6-7 | 0 | 22 | 22 | 22 | 22 | 0 ✓ | 0 | 0 |
| 6-8 | 8 | 22 | 30 | 24 | 32 | 2 | 2 | 2 |
| 7-8 | 10 | 22 | 32 | 22 | 32 | 0 ✓ | 0 | 0 |
| 8-9 | 6 | 32 | 38 | 32 | 38 | 0 ✓ | 0 | 0 |

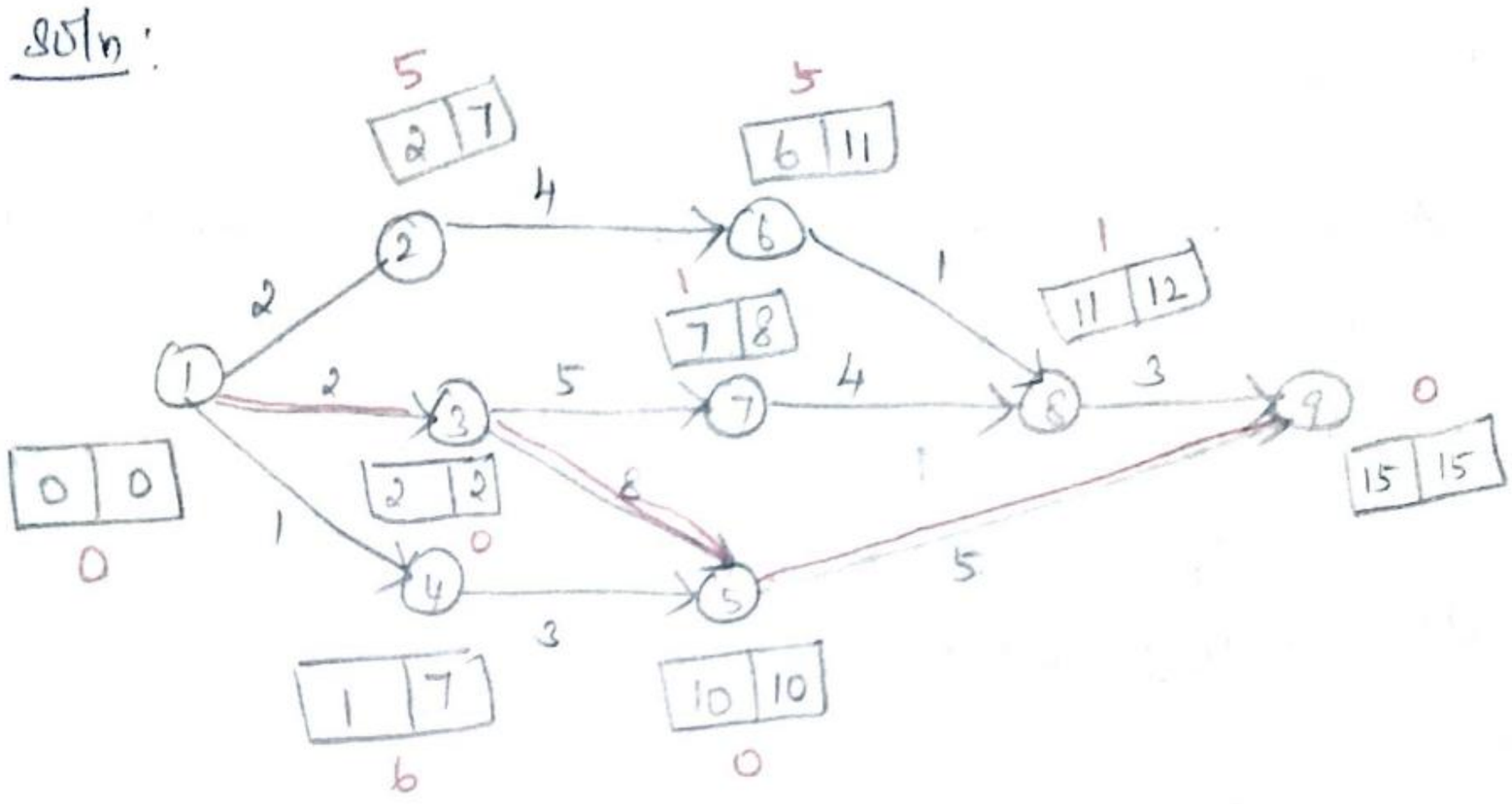
Critical path = 1-2-4-6-7-8-9

Project Duration = 38 hrs.

4 Find the critical path and slack time!




Soln:




Float Table:

| Activity | T | EST | EFT | LST | LFT | Float | slack |
|---|---|---|---|---|---|---|---|
| 1-2 | 2 | 0 | 2 | 5 | 7 | 3 | 5 |
| (1-3) | 2 | 0 | 2 | 0 | 2 | 0 ✓ | 0 |
| 1-4 | 1 | 0 | 1 | 6 | 7 | 5 | 6 |
| 2-6 | 4 | 2 | 6 | 7 | 11 | 5 | 5 |
| (3-5) | 8 | 2 | 10 | 2 | 10 | 0 ✓ | 0 |
| 3-7 | 5 | 2 | 7 | 3 | 8 | 1 | 1 |
| 4-5 | 3 | 1 | 4 | 7 | 10 | 6 | 6 |
| (5-9) | 5 | 10 | 15 | 10 | 15 | 0 ✓ | 0 |
| 6-8 | 1 | 6 | 7 | 11 | 12 | 5 | 5 |
| 7-8 | 4 | 7 | 11 | 8 | 12 | 1 | 1 |
| 8-9 | 3 | 11 | 14 | 12 | 15 | 1 | 1 |

critical path = 1-3-5-9

Project duration = 15 days.

## Homework :

1. $A < D$, $A < E$, $B < F$, $D < F$, $C < G$, $C < H$, $F < I$, $G < I$.

2. $A < D, E$ ; $B, D < F$ ; $C < G$ ; $B, G < H$ ; $F, G < I$.

3. (i) A and B are starting jobs, (ii) A controls C, D, E (iii). B controls F, K. (iv). G depends on C (v). H depends on D (vi). E, F controls J and M (vii). L depends on K (viii) M is also controlled by L (ix) G, H, J and M are the last jobs.

4. Event no : 1 2,3 4 5 6 7
   Proceded by : — 1 2,3 3 4,5 5,6.

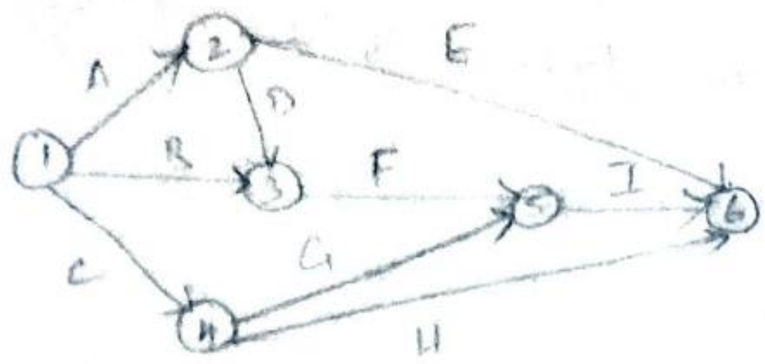
5. Activity : A B C D E F G
   Immediate Predecessor : — — A C D B,E E,F.

6. Activity : A B C D E F
   Immediate Predecessor : — A A B,C — E.

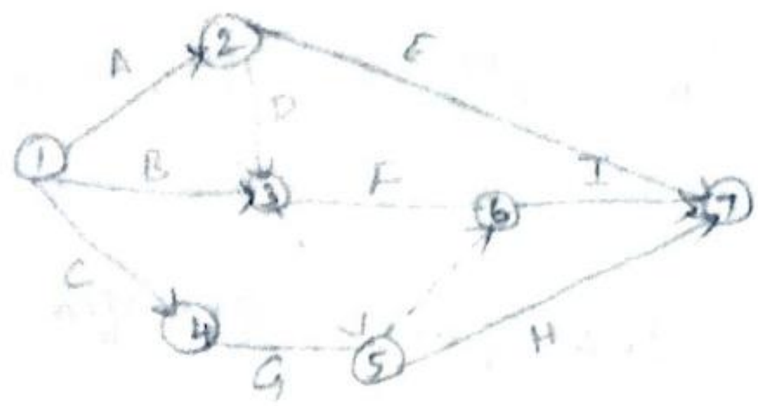
7. Activity : A B C D E F G H I.
   Immediate predecessor : — — — A B C D,E B H,F.

8. Activity : A B C D E F G
   Preceded by : — — — A,B A,B C,D,E C,D,E.

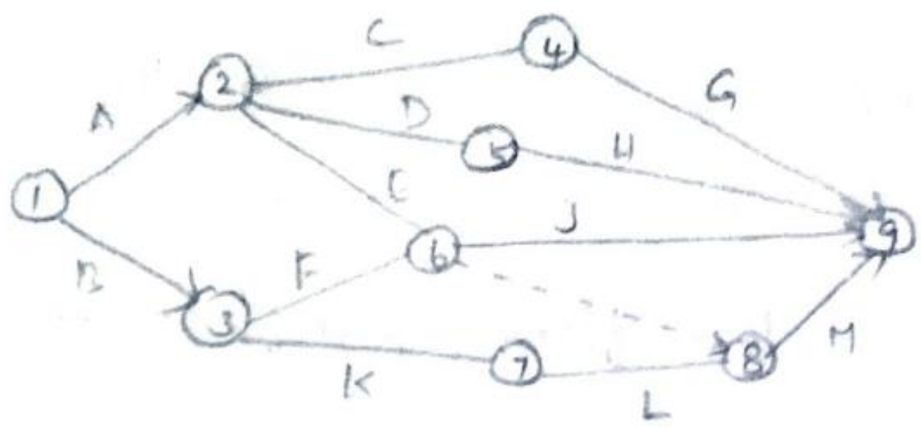
1)



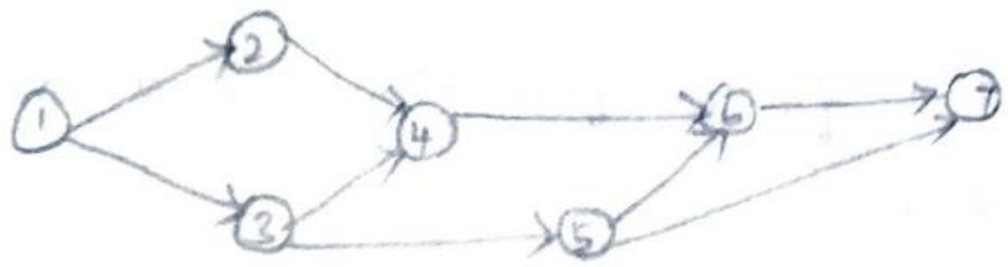

2)



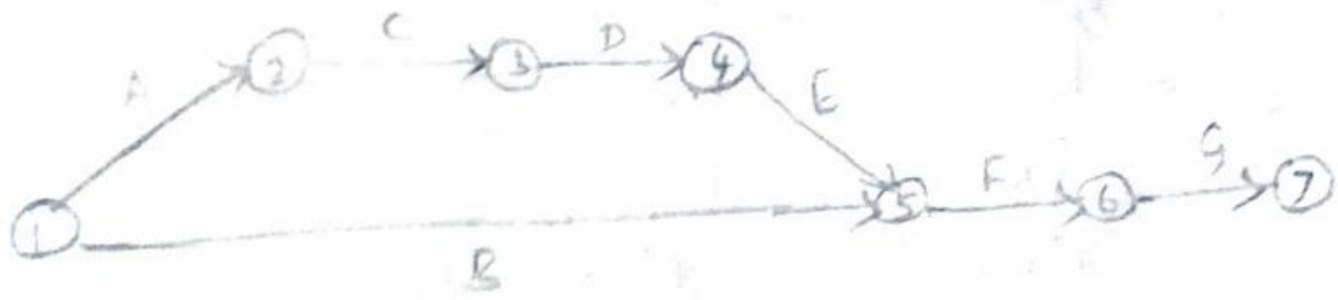

3)



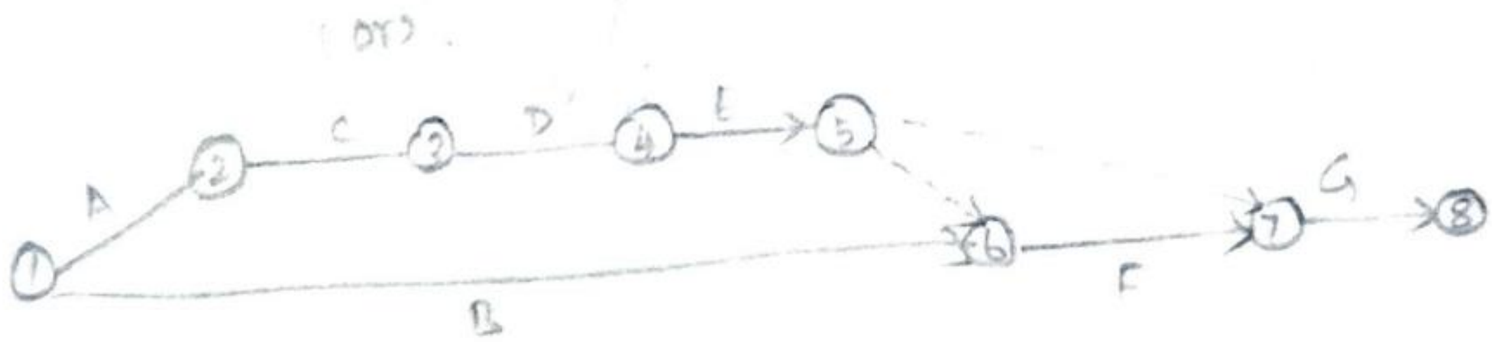

4)



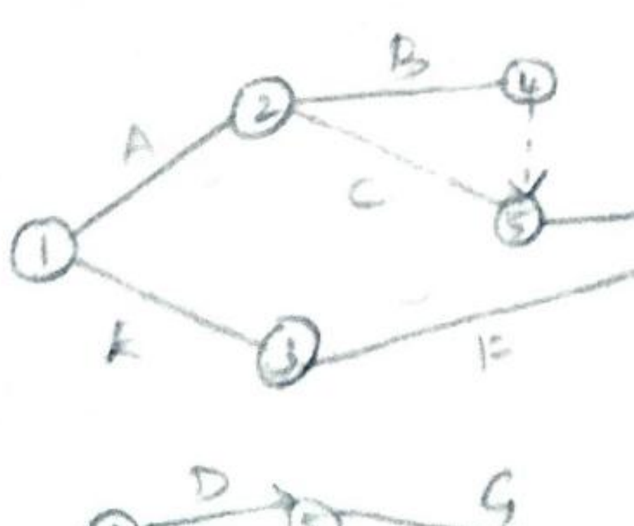
5)



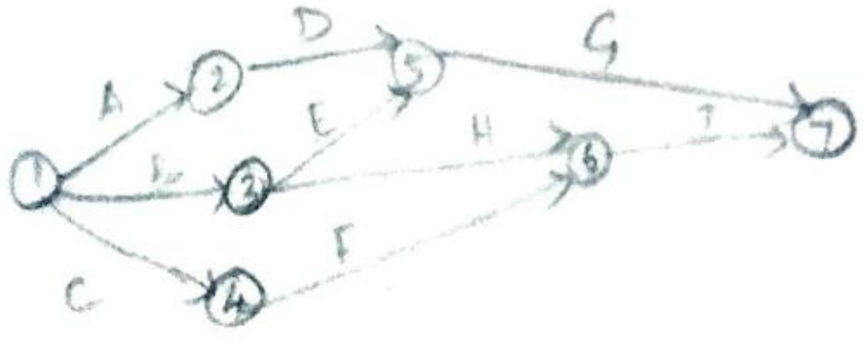

(OR)



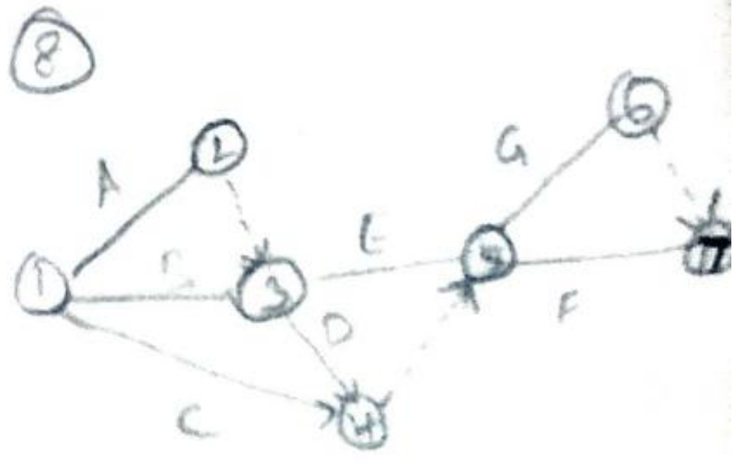

6)




7)




8)



## Homework: Critical Path method

1. Act: 1-2 1-3 2-4 3-4 4-5 4-6 5-7 6-7 7-8

Time: 5 4 6 2 1 7 8 4 3

find the critical path & project duration.

2. Act: 1-2 1-3 1-4 2-5 3-6 3-7 4-6 5-8 6-9 7-8 8-9

Time (weeks): 2 2 1 4 8 5 3 1 5 4 3

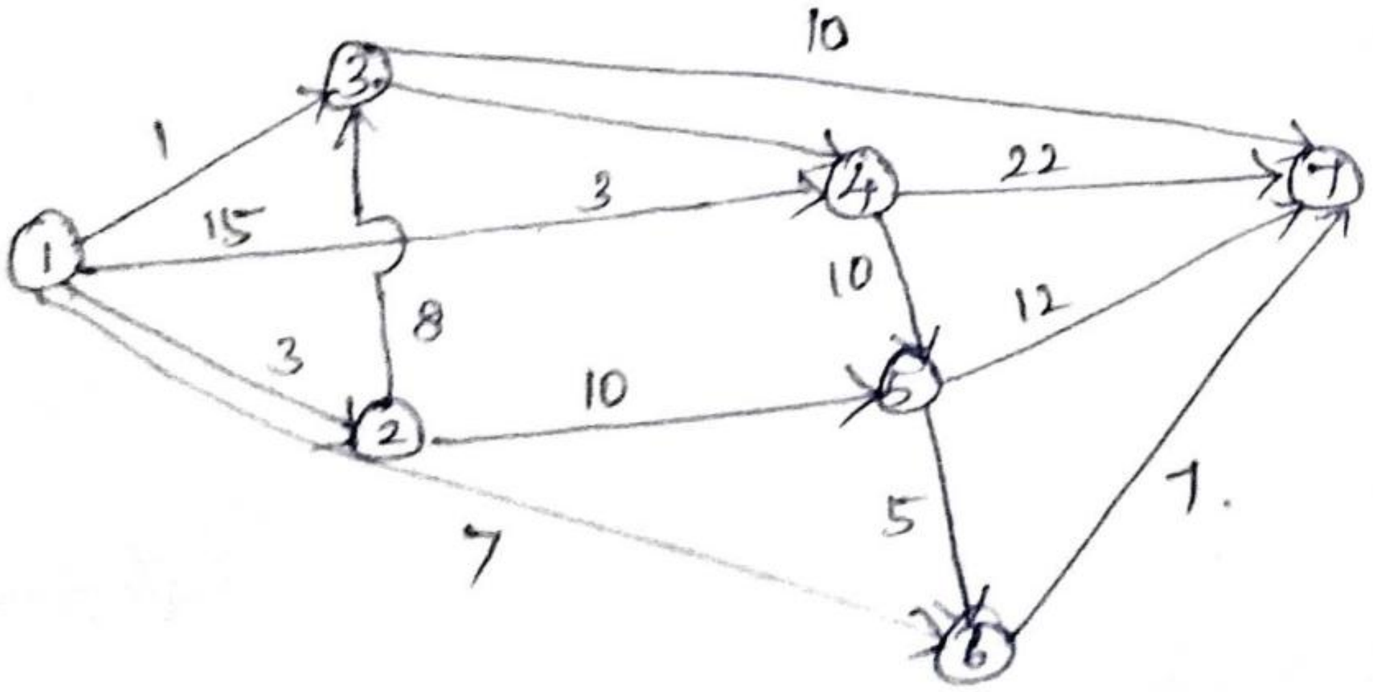
Find the critical path, Total float, Free float.

3 Act: 0-1 1-2 1-3 2-4 2-5 3-4 3-6 4-7 5-7 6-7

Duration: 2 8 10 6 3 3 7 5 2 8

Find critical path, total float, free float and independent float.

4. Calculate the total float, free floats and summarize the critical path.



PERT

1. 

| Job: | 1-2 | 1-6 | 2-3 | 2-4 | 3-5 | 4-5 | 6-7 | 5-8 | 7-8 |
|---|---|---|---|---|---|---|---|---|---|
| optimistic | 3 | 2 | 6 | 2 | 5 | 3 | 3 | 1 | 4 |
| Most likely | 6 | 5 | 12 | 5 | 11 | 6 | 9 | 4 | 19 |
| Pessimistic | 15 | 14 | 30 | 8 | 17 | 15 | 27 | 7 | 28 |

(i) Find the probability of completing the project before 31 days.

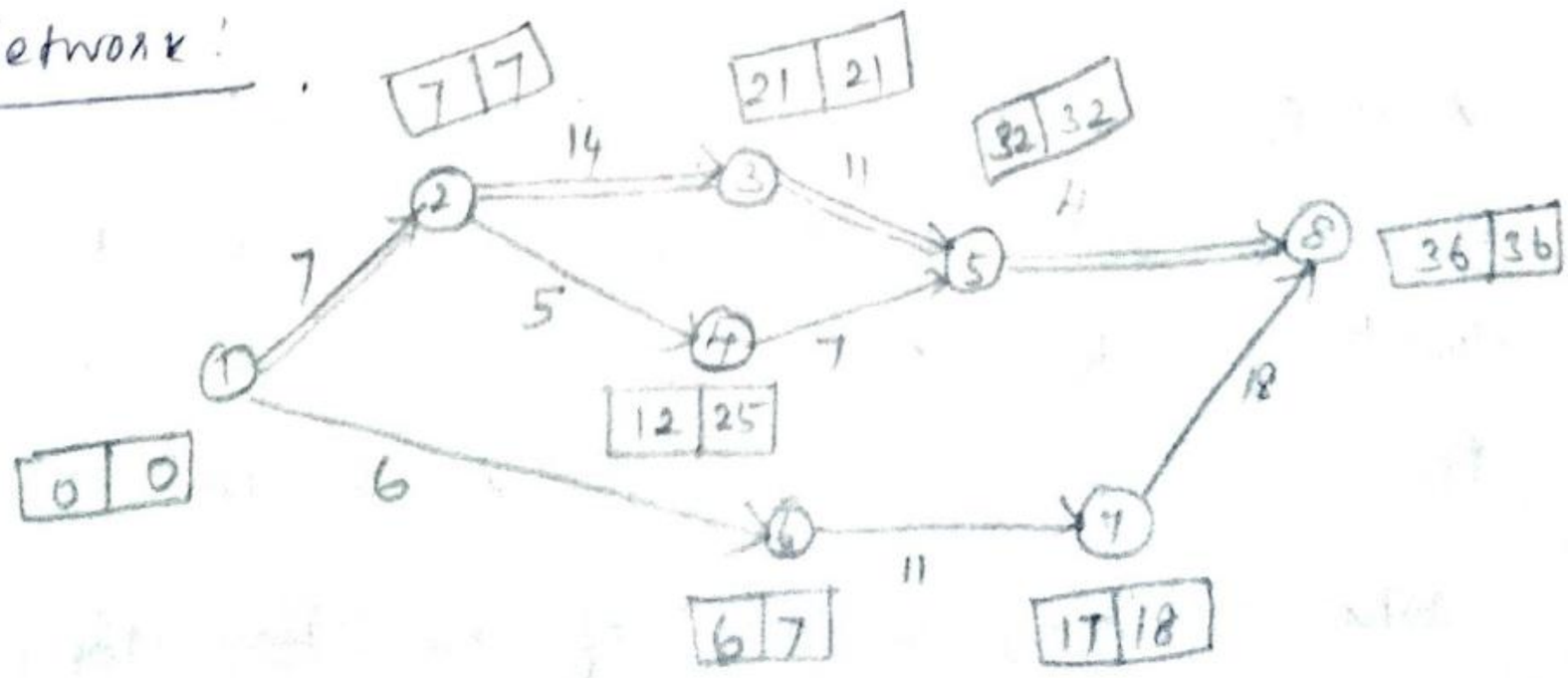
(ii). What is the chance that the project duration exceeding 46 days.

Soln:

Expected time and variance Table:

| Act | to | tm | tp | $te = \frac{to+4tm+tp}{6}$ | $\sigma^2 = \left(\frac{tp-to}{6}\right)^2$ |
|---|---|---|---|---|---|
| 1-2 | 3 | 6 | 15 | 7 | 4 ✓ |
| 1-6 | 2 | 5 | 14 | 6 | 4 |
| 2-3 | 6 | 12 | 30 | 14 | 16 ✓ |
| 2-4 | 2 | 5 | 8 | 5 | 1 |
| 3-5 | 5 | 11 | 17 | 11 | 4 ✓ |
| 4-5 | 3 | 6 | 15 | 7 | 4 |
| 6-7 | 3 | 9 | 27 | 11 | 16 |
| 5-8 | 1 | 4 | 7 | 4 | 1 ✓ |
| 7-8 | 4 | 19 | 28 | 18 | 16 |

Network:



Critical path = 1-2-3-5-8

Project Duration = 36 days

Variance of the project length = 4 + 16 + 4 + 1

= 25

(i) The probability of completing the project with in 31 days

Standard normal deviate $Z = \frac{\text{Due date} - \text{Expected date}}{\sqrt{\text{variance of the project length}}}$

$= \frac{31 - 36}{\sqrt{25}} = \frac{-5}{5} = -1.$

$P(t \leq 31) \equiv P(Z \leq -1.0) = 0.1587$

$= 15.87\%$

(ii) The probability of completing the project with in 46 days.

Standard normal deviate $Z = \frac{\text{Due date} - \text{Expected date}}{\sqrt{\text{Variance}}}$

$= \frac{46 - 36}{\sqrt{25}} = \frac{10}{5} = 2.$

$P(t \leq 46) = P(Z \leq 2) = 0.9773$

$= 97.73\ \%.$

2. Activity : 1-2 1-3 2-3 2-4 2-5 3-4 4-7 5-6 5-7 6-7

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Optimistic : | 3 | 1 | 6 | 0 | 2 | 3 | 6 | 1 | 2 | 4 |
| Most likely : | 4 | 2 | 8 | 0 | 5 | 5 | 9 | 1 | 5 | 8 |
| Pessimistic : | 5 | 3 | 10 | 0 | 8 | 7 | 12 | 1 | 8 | 12 |

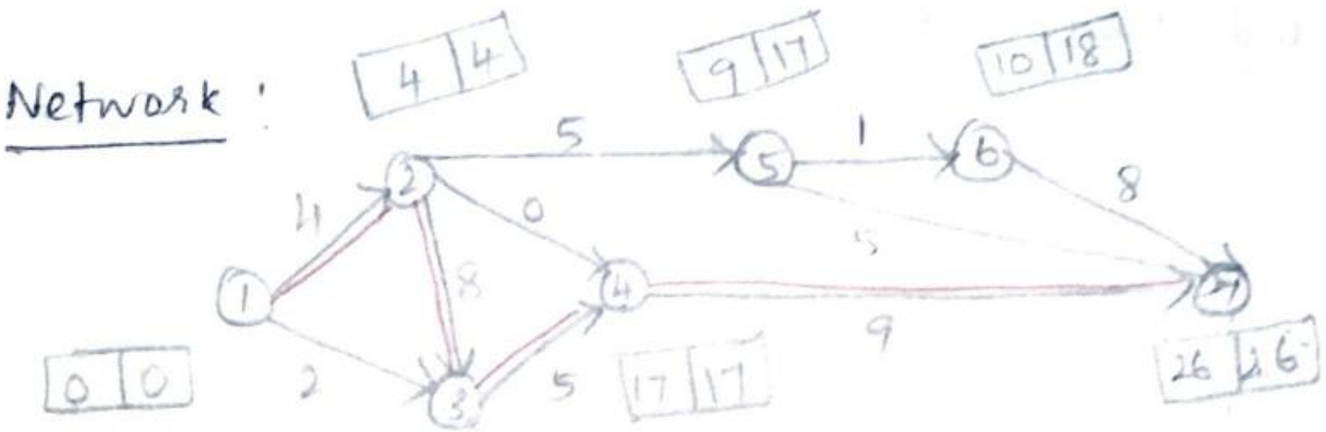
What is the probability of completing the project with in (i) 24 days (ii) 28 days.

Soln:

Expected Time and float Table:

| Act. | to | tm | tp | $te = \frac{to+4tm+tp}{6}$ | $\sigma^2 = \left(\frac{tp-to}{6}\right)^2$ |
|---|---|---|---|---|---|
| 1-2 | 3 | 4 | 5 | 4 | 0.111 ✓ |
| 1-3 | 1 | 2 | 3 | 2 | 0.111 |
| 2-3 | 6 | 8 | 10 | 8 | 0.445 ✓ |
| 2-4 | 0 | 0 | 0 | 0 | 0 |
| 2-5 | 2 | 5 | 8 | 5 | 1.000 |
| 3-4 | 3 | 5 | 7 | 5 | 0.445 ✓ |
| 4-7 | 6 | 9 | 12 | 9 | 1 ✓ |
| 5-6 | 1 | 1 | 1 | 1 | 0 |
| 5-7 | 2 | 5 | 8 | 5 | 1 |
| 6-7 | 4 | 8 | 12 | 8 | 1.778 |

Network:




Critical Path = 1-2-3-4-7.

Project duration = 26 days.

Variance of the project length = 0.111 + 0.445 + 0.445 + 1

= 2.001

(i) The prob of completing the project with in 24 days.

$$\text{Standard normal deviate } z = \frac{\text{Due date} - \text{Expected date}}{\sqrt{\text{Variance}}}$$

$$= \frac{24-26}{\sqrt{2.001}} = \frac{-2}{1.414} = -1.414$$

$$P(t \le 24) = P(z \le -1.414) = 0.0787 = 7.87\ \%$$

(ii) The prob of completing the project with in 28 days

$$z = \frac{\text{Due date} - \text{Expected date}}{\sqrt{\text{Variance}}} = \frac{28-26}{\sqrt{2.001}} = \frac{2}{\sqrt{2.001}} = 1.414$$

$P(t \leq 28) = P(z \leq 1.414)$

$= 0.9213$

$= 92.13\ \%$

3. Act : 1-2 2-3 2-4 3-5 4-5 4-6 5-7 6-7 7-8 7-9

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| most likely: | 2 | 2 | 3 | 4 | 3 | 5 | 5 | 7 | 4 | 6 |
| optimistic: | 1 | 1 | 1 | 3 | 2 | 3 | 4 | 6 | 2 | 4 |
| Pessimistic: | 3 | 3 | 5 | 5 | 4 | 7 | 6 | 8 | 6 | 8 |

| 8-10 | 9-10 |
|---|---|
| 2 | 5 |
| 1 | 3 |
| 3 | 7 |

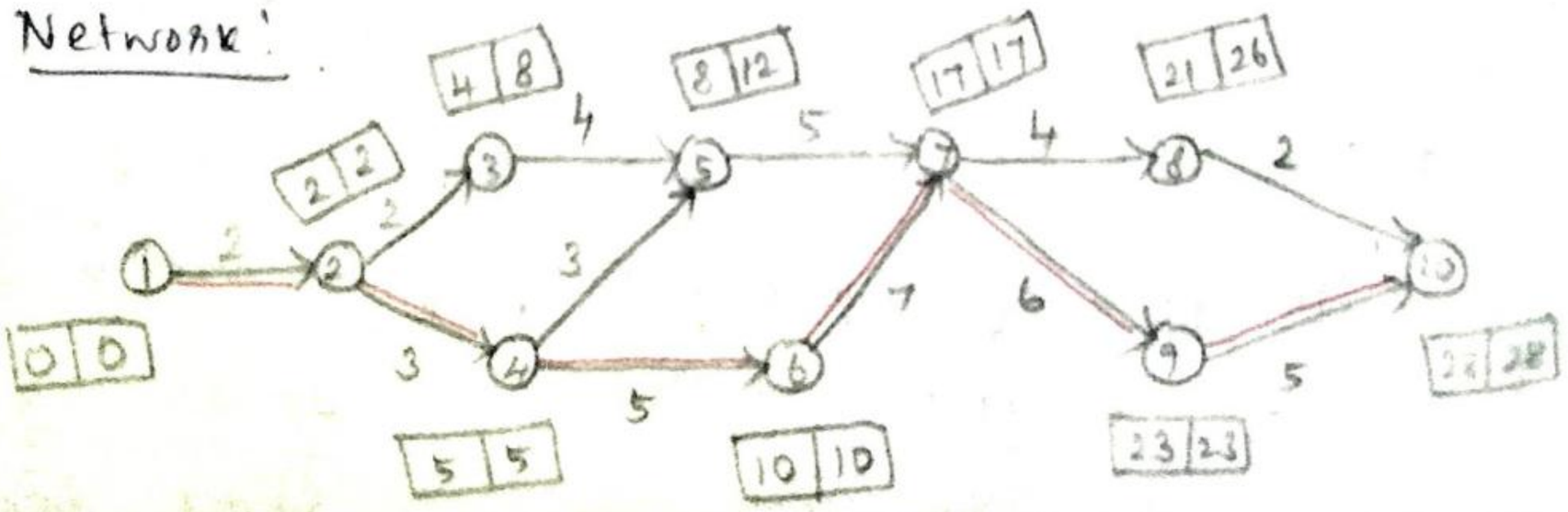
(i) what is the probability of Completing the project in 30 days

Soln:

Expected time and variance Table:

| Act | to | tm | tp | $te = \frac{to+4tm+tp}{6}$ | $\sigma^2 = \left(\frac{tp-to}{6}\right)^2$ |
|---|---|---|---|---|---|
| 1-2 | 1 | 2 | 3 | 2 | 0.111 ✓ |
| 2-3 | 1 | 2 | 3 | 2 | 0.111 |
| 2-4 | 1 | 3 | 5 | 3 | 0.445 ✓ |
| 3-5 | 3 | 4 | 5 | 4 | 0.111 |
| 4-5 | 2 | 3 | 4 | 3 | 0.111 |
| 4-6 | 3 | 5 | 7 | 5 | 0.445 ✓ |
| 5-7 | 4 | 5 | 6 | 5 | 0.111 |
| 6-7 | 6 | 7 | 8 | 7 | 0.111 ✓ |
| 7-8 | 2 | 4 | 6 | 4 | 0.445 |
| 7-9 | 4 | 6 | 8 | 6 | 0.445 ✓ |
| 8-10 | 1 | 2 | 3 | 2 | 0.111 |
| 9-10 | 3 | 5 | 7 | 5 | 0.445 ✓ |

Network:



Critical path = 1-2-4-6-7-9-10

Project duration = 28 days

Variance of the Project length = 0.111 + 0.445 + 0.445 + 0.111 + 0.445 + 0.445

= 2.002

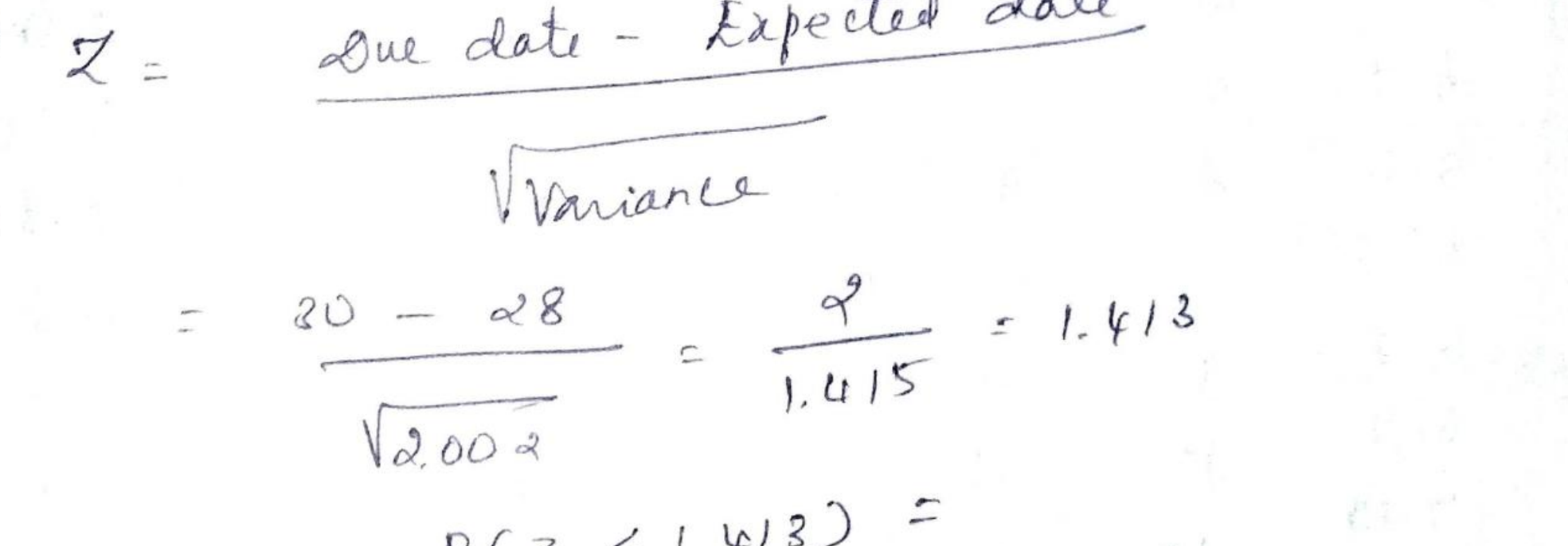
(i) The prob that the project will be completed with in 30 days

$$Z = \frac{\text{Due date} - \text{Expected date}}{\sqrt{\text{Variance}}}$$

$$= \frac{30 - 28}{\sqrt{2.002}} = \frac{2}{1.415} = 1.413$$

$P(t \leq 30) = P(z \leq 1.413) =$

4.



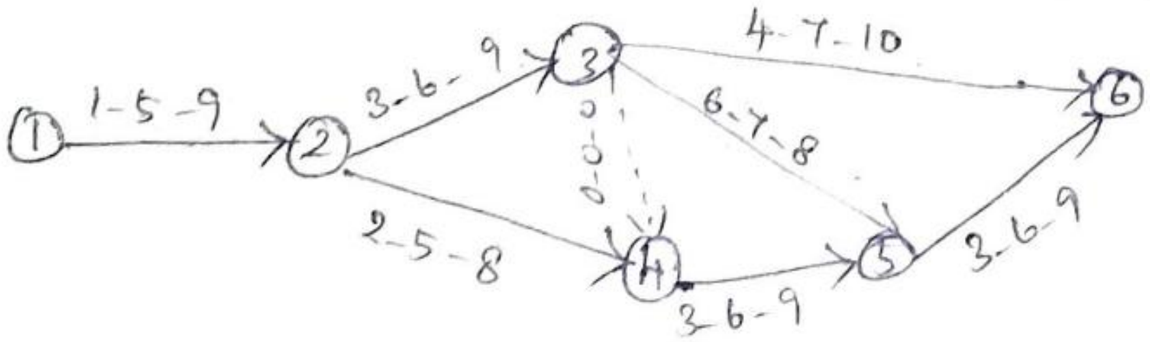

Find the ~~Critical path~~. Expected time and variance.

Soln:

Expected Time and Variance Table:

| Act | to | tm | tp | $te = \frac{to+4tm+tp}{6}$ | $\sigma^2 = \left(\frac{tp-to}{6}\right)^2$ |
|---|---|---|---|---|---|
| 1-2 | 3 | 6 | 10 | 6.2 | 1.36 |
| 1-3 | 6 | 7 | 12 | 7.7 | 1 |
| 1-4 | 7 | 9 | 12 | 9.2 | 0.69 |
| 2-3 | 0 | 0 | 0 | 0 | 0 |
| 2-5 | 8 | 12 | 17 | 12.2 | 2.25 |
| 3-6 | 10 | 12 | 15 | 12.2 | 0.69 |
| 4-7 | 8 | 13 | 19 | 13.2 | 3.36 |
| 5-6 | 8 | 9 | 10 | 9 | 0.11 |
| 5-8 | 12 | 14 | 15 | 13.8 | 0.25 |
| 6-7 | 0 | 0 | 0 | 0 | 0 |
| 6-9 | 13 | 16 | 19 | 16 | 1 |
| 7-10 | 10 | 13 | 17 | 13.2 | 1.36 |
| 8-9 | 4 | 7 | 10 | 7 | 1 |
| 9-11 | 6 | 8 | 12 | 8.3 | 1 |
| 10-11 | 10 | 12 | 14 | 12 | 0.44 |

5.

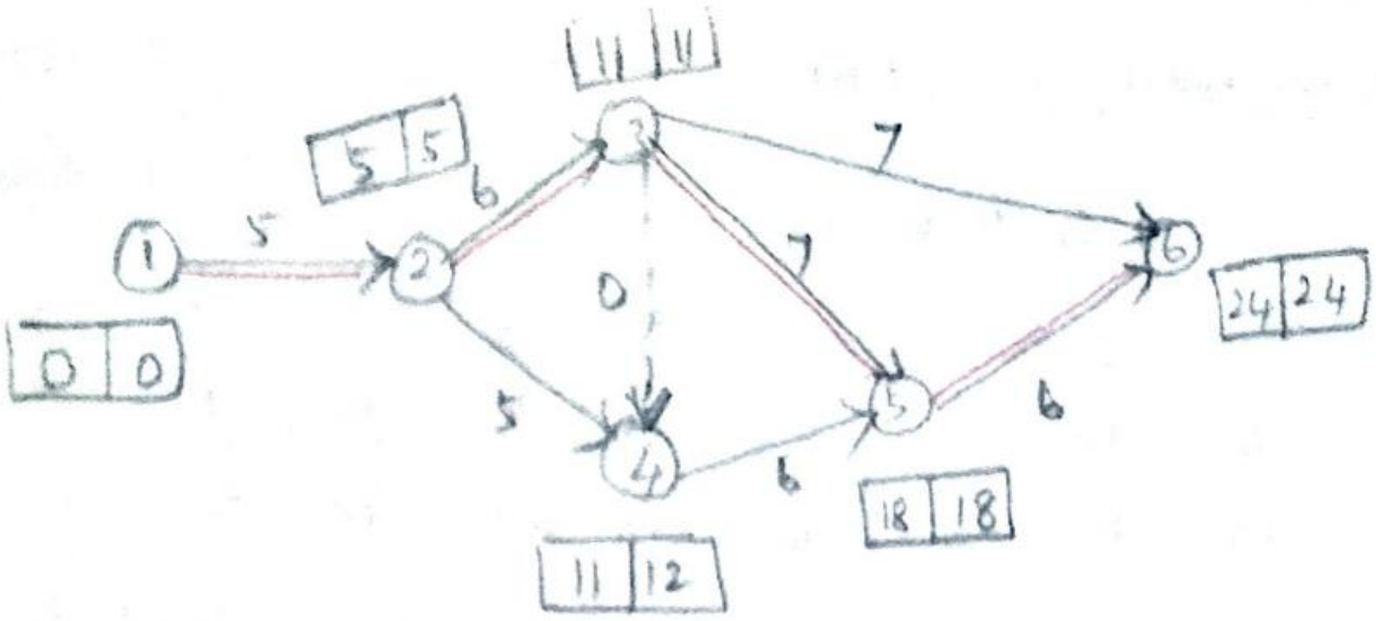



Find the prob of completing the project in (i) 21 days
(ii) 19 days.

Soln:

Expected Time and Variance Table:

| Act | to | tm | tp | $te = \frac{to+4tm+tp}{6}$ | $\sigma^2 = \left(\frac{tp-to}{6}\right)^2$ |
|---|---|---|---|---|---|
| 1-2 | 1 | 5 | 9 | 5 | 1.78 ✓ |
| 2-3 | 3 | 6 | 9 | 6 | 1 ✓ |
| 2-4 | 2 | 5 | 8 | 5 | 1 |
| 3-4 | 0 | 0 | 0 | 0 | 0 |
| 3-5 | 6 | 7 | 8 | 7 | 0.11 ✓ |
| 3-6 | 4 | 7 | 10 | 7 | 1 |
| 4-5 | 3 | 6 | 9 | 6 | 1 |
| 5-6 | 3 | 6 | 9 | 6 | 1 ✓ |




Critical path = 1-2-3-5-6

Project duration = 24 days

Variance of the project length = 1.078 + 1 + 0.11 + 1

= 3.089

(i). Prob of completing the project with in 21 days

$$z = \frac{\text{Due date} - \text{Expected date}}{\sqrt{\text{variance}}}$$

$$= \frac{21 - 24}{\sqrt{3.89}} = \frac{-3}{1.972} = -1.521$$

$$P(t \le 21) = P(z \le -1.521)$$

$$= 0.0641$$

$$= 6.41\%$$

(ii) Prob of completing the project with in 19 days

$$z = \frac{\text{Due date} - \text{Expected date}}{\sqrt{\text{variance}}}$$

$$= \frac{19 - 24}{1.972} = \frac{-5}{1.972} = -2.535$$

$$P(t \le 19) = P(z \le -2.535)$$

$$= 0.0056$$

$$= 0.56\%$$

## Home Work: PERT

1. Act: 1-2 1-3 2-4 3-5 3-6 4-5 4-8 5-6 5-8 6-8

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| a: | 1 | 4 | 1 | 2 | 4 | 3 | 6 | 1 | 6 | 2 |
| m: | 4 | 10 | 4 | 5 | 7 | 6 | 15 | 4 | 9 | 8 |
| b: | 19 | 16 | 7 | 14 | 16 | 21 | 30 | 7 | 18 | 26 |

Find the prob of the project will be completed in 38 weeks

2. Act: 1-2 1-3 2-4 2-5 3-4 3-6 4-5 4-6 5-7 6-7

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Expected Time (months): | 4 | 5 | 2 | 12 | 3 | 8 | 10 | 6 | 8 | 10 |
| Variance: | 8 | 3 | 1 | 5 | 2 | 4 | 4 | 2 | 1 | 8 |

Find the prob of completing project with in 32 months.

3. Act: 1-2 1-3 1-4 2-5 3-5 4-6 5-6

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| to: | 1 | 1 | 2 | 1 | 2 | 2 | 3 |
| tm: | 1 | 4 | 2 | 1 | 5 | 5 | 6 |
| tp: | 7 | 7 | 8 | 1 | 14 | 8 | 15 |

What is the prob that the project will be completed
(i) at least 4 weeks earlier than expected Time
(ii) no more than 4 weeks later than expected Time

4. Act: 1-2 1-3 1-4 2-3 2-5 3-4 3-6 4-6 5-6

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| a: | 2 | 6 | 6 | 2 | 11 | 15 | 3 | 9 | 4 |
| b: | 4 | 6 | 12 | 5 | 14 | 24 | 6 | 15 | 10 |
| c: | 6 | 6 | 24 | 8 | 23 | 45 | 9 | 27 | 16 |

a) optimistic b) most likely c) pessimistic time

What is the prob that the project will be completed in 60 days.