Newspaper Boy Problem: Unit - V.

1. A newspaper boy buys paper for 60 paise each and sells them for Rs. 1.40 each. He cannot return unsold newspapers. Daily demand has the following distribution.

No. of customers	23	24	25	26	27	28	29	30	31	32
Probability	.01	.03	.06	.10	.20	.25	.15	.10	.05	.05

If each day's demand is independent of the previous day's, how many paper should be ordered each day?

Solution:

$$C_1 = 60 \text{ paise} = Rs.\ 0.60$$

$$C_2 = 1.40 - 0.60 = Rs.\ 0.80$$

$$\frac{C_2}{C_1 + C_2} = \frac{0.80}{0.60 + 0.80} = 0.57$$

No. of customers.	Probability	Cumulative Probability
23	.01	.01
24	.03	.04
25	.06	.10
26	.10	.20
27	.20	.40
28	.25	.65
29	.15	.80
30	.10	.90
31	.05	.95
32	.05	1.

$$.40 < \frac{C_2}{C_1 + C_2} < .65$$

$$\therefore Q = 28$$

Optimum number of news papers to be ordered is 28.

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2. A newspaper boy buys papers for Rs. 1.40 and sells them for Rs. 2.45 each. He cannot return unsold newspapers. Daily demand has the following distribution:

Customers:	25	26	27	28	29	30	31	32	33	34	35	36
Probability:	.03	.05	.05	.10	.15	.15	.12	.10	.10	.07	.06	.02

If each day's demand is independent of the previous days how many papers he should order each day?

Solution:

$$C_1 = Rs.\ 1.40.$$

$$C_2 = 2.45 - 1.40 = Rs.\ 1.05$$

$$\frac{C_2}{C_1+C_2} = \frac{1.05}{1.40+1.05} = \frac{1.05}{2.45} = 0.43.$$

No. of customers	Probability	Cumulative Probability
25	.03	.03
26	.05	.08
27	.05	.13
28	.10	.23
29	.15	.38
30	.15	.53
31	.12	.65
32	.10	.75
33	.10	.85
34	.07	.92
35	.06	.98
36.	.02	1

$$0.38 < \frac{C_2}{C_1+C_2} < 0.53.$$

$$Q = .30$$

Optimum number of newspapers to be ordered is 30

A probability distribution of monthly sales of an item is as follows

Monthly Sales :	0	1	2	3	4	5	6
probability :	.01	.06	.25	.30	.22	.10	.06

The cost of carrying inventory is Rs. 30/ unit /month cost of unit shortage is Rs. 70. Determine optimum stock to minimize expected cost.

$C_1 = Rs.\ 30$

$C_2 = Rs\ 70$

$$\frac{C_2}{C_1 + C_2} = \frac{70}{30 + 70} = 0.70$$

Monthly sales	Probability	Cumulative Probability
0	.01	.01
1	.06	.07
2	.25	.32
3	.30	.62
4	.22	.84
5	.10	.94
6	.06	1

$$0.62 < \frac{C_2}{C_1 + C_2} < 0.84$$

$\therefore$ optimum stock = 4. units

4. The probability distribution of the demand for a product has been estimated to be:

Demand:	0	1	2	3	4	5	6
Probability:	.05	.15	.30	.35	.10	.05	0

Each unit sells for Rs.100 and the ~~to~~ total incremental cost per unit sold is Rs. 60.

If the product is not sold, It is completely worthless. The purchase cost of a unit is Rs. 10. How many units should purchase.

$$C_1 = Rs.\ 10.$$

$$C_2 = 100 - 60 = Rs.\ 40.$$

$$\frac{C_2}{C_1 + C_2} = \frac{40}{10+40} = 0.8.$$

Demand	Probability	Cumulative Probability
0	.05	.05
1	.15	.20
2	.30	.50
3	.35	.85
4	.10	.95
5	.05	1
6	0	1

$$\therefore .50 < \frac{C_2}{C_1 + C_2} < 0.85.$$

Optimum purchase quantity $Q = 3$ units.

6. The probability distribution of monthly sales of a certain item is as follows:

monthly sales:	0	1	2	3	4	5	6
Probability:	.02	.05	.30	.27	.20	.10	.06

The cost of carrying inventory is Rs. 10 per unit per month. The current policy is to maintain a stock of 4 items at the begining of each month. Assuming that the cost of shortage is proportional to both time and quantity sort. obtain the imputed cost of a shortage ~~cost~~ of one item for one time unit.

The least value of C_2 can be determined by

$$\frac{C_2}{C_1+C_2} = \sum_{x=0}^{Q-1} p(x) + \left(Q-\frac{1}{2}\right)\sum_{x=Q}^{6}\frac{p(x)}{x}$$

$$= \sum_{x=0}^{3} p(x) + \left(4-\frac{1}{2}\right)\sum_{x=4}^{6}\frac{p(x)}{x}$$

$$= p(0)+p(1)+p(2)+p(3)+\frac{7}{2}\left[\frac{p(4)}{4}+\frac{p(5)}{5}+\frac{p(6)}{6}\right]$$

$$= .02+.05+.30+.27+\frac{7}{2}\left[\frac{.20}{4}+\frac{.10}{5}+\frac{.06}{6}\right]$$

$$= 0.64+\frac{7}{2}\left[.05+.02+.01\right]$$

$$= 0.64+\frac{7}{2}\left[.08\right]$$

$$= 0.64+7\left[.04\right]$$

$$= 0.64+.28 = 0.92$$

$$\frac{C_2}{10+C_2} = 0.92$$

$$C_2 = 0.92\,(10+C_2)$$

$$C_2 = 9.2+0.92\,C_2$$

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$$C_2 - 0.92 C_2 = 9.2$$

$$C_2(1 - 0.92) = 9.2$$

$$C_2(0.08) = 9.2$$

$$C_2 = \frac{9.2}{0.08}$$

$$C_2 = 115$$

The greatest value of C_2 can be determined by.

$$\frac{C_2}{C_1 + C_2} = \sum_{x=0}^{Q} p(x) + \left(Q + \frac{1}{2}\right) \sum_{x=Q+1}^{6} \frac{p(x)}{x}$$

$$= \sum_{x=0}^{4} p(x) + \left(4 + \frac{1}{2}\right) \sum_{x=5}^{6} \left[\frac{p(x)}{x}\right]$$

$$= P(0) + P(1) + P(2) + P(3) + P(4) + \frac{9}{2}\left[\frac{P(5)}{5} + \frac{P(6)}{6}\right]$$

$$= .02 + .05 + .30 + .27 + .20 + \frac{9}{2}\left[\frac{.10}{5} + \frac{.06}{6}\right]$$

$$= .84 + \frac{9}{2}\left[.02 + .01\right]$$

$$= .84 + \frac{9}{2}\left[.03\right]$$

$$= .84 + 0.135$$

$$\frac{C_2}{10 + C_2} = .975$$

$$C_2 = 0.975(10 + C_2)$$

$$= 9.75 + .975 C_2$$

$$C_2 - 0.975 C_2 = 9.75$$

$$C_2(1 - 0.975) = 9.75$$

$$C_2(0.025) = 9.75$$

$$C_2 = \frac{9.75}{0.025} = 390$$

$\therefore$ The shortage cost $115 < C_2 < 390$

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Definition: Price break.

The unit cost of an item is independent of the quantity procured. Often, discounts are offered for the purchase of large quantities, These discounts take the form of price breaks.

$$Q = \sqrt{\frac{2DC_3}{C_1}}$$

and $T_C = DK_1 + \frac{D}{Q} C_3 + \frac{1}{2} Q K_1 I$.

EOQ Problem with one price break:

When there is only one price break, the situation may be illustrated as follows:

Range of quantity.	Purchase cost per unit.
$0 \leq Q_1 < b$	K_{11}
$b \leq Q_2$.	K_{12}.

where b is that quantity at and beyond which the quantity discount applies and $K_{12} < K_{11}$.

Step: 1

Compute Q_2^*, optimum order quantity for the lowest price and compare it with the quantity b.

If $Q_2^* \geq b$, then place orders for quantities of size Q_2^* and obtain discount otherwise go to the next step.

Step: 2

If $Q_2^* < b$, we cannot place order at the reduced price K_{12}.

$\therefore$ In order to obtain the optimum order quantity, we need only to compare the total inventory cost for $Q = Q_1^*$ with $Q = b$.

The values of $T_c(Q_1^*)$ and $T_c(b)$ may be determined as follows:

$$T_c(Q_1^*) = DK_{11} + \frac{D}{Q_1^*} C_s + \frac{1}{2} Q_1^* \times K_{11} \times I.$$

$$T_c(b) = DK_{12} + \frac{D}{b} C_s + \frac{1}{2} b \times K_{12} \times I.$$

If $T_c(Q_1^*) > T_c(b)$, then

$$Q^* = b$$

otherwise, $Q^* = Q_1^*$.

EOQ problem with two price breaks:

When there are two price breaks, the situation may be illustrated as follows:

Range of quantity	Purchase cost per unit.
$0 \leq Q_1 < b_1$	K_{11}
$b_1 \leq Q_2 < b_2$	K_{12}
$b_2 \leq Q_3$.	K_{13}.

Where b_1 and b_2 are those quantities which determine the price breaks.

Step: 1

Compute Q_3^* and compare it with b_2.

Step: 2

If $Q_3^* \geq b_2$, the optimum order quantity is Q_3^*.

If $Q_3^* < b_2$, proceed on to next step.

Step: 3

Compute Q_2^*. Now, since $Q_3^* < b_2$, therefore $Q_2^* < b_2$,

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because, $Q_1^* < Q_2^* < \ldots < Q_n^*$.

Thus, either $Q_2^* < b_1$ (or) $b_1 \le Q_2^* < b_2$.

Step: 4

If $Q_3^* < b_2$ and $b_1 \le Q_2^* < b_2$, the proceed as in the case of only one price break, that is, compare $T_C(Q_2^*)$ and $T_C(b_2)$ to determine the optimum order quantity.

Step: 5

If $Q_2^* < b_2$ and $Q_2^* < b_1$, then compute Q_1^* which will now satisfy the inequality $Q_1 < b_1$.

Compare $T_C(Q_1^*)$ with $T_C(b_1)$ and $T_C(b_2)$ so as to get the optimum purchase quantity.

1. Find the optimum order quantity for a product for which the price break as follows:

Quantity	unit cost.
$0 \le Q_1 < 500$	10
$500 \le Q_2$	9.25.

The monthly demand is product is 200 units the cost of storage is 2% of the unit cost and the cost of ordering is Rs 350.

Solution:

Given. $D = 200$ units /month.

$C_S = Rs.\ 350$.

$I = 2\% = \frac{2}{100} = Rs.\ 0.02$.

$K_{11} = Rs.\ 10$

$K_{12} = Rs.\ 9.25$

$b = 500$.

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$$Q_2^* = \sqrt{\frac{2DC_s}{k_{12}I}}$$

$$= \sqrt{\frac{2 \times 200 \times 350}{9.25 \times 0.02}}.$$

$$= 870 \text{ units}.$$

Q_2^* satisfies $500 \leq Q_2$.

$\therefore$ optimum order quantity $Q = Q_2^*$

$Q = 870$ units.

2. Find the optimum order quantity for a product for which the pricebreaks are as follows:

Quantity	unit cost
$0 \leq Q_1 < 800$	1
$800 \leq Q_2$	0.98

The yearly demand for the product is 1600 units, cost of placing an order is Rs. 5. The cost of storage is 10 % per year.

Given. $D = 1600$ units / year.

$C_s =$ Rs. 5

$I = 10\% =$ Rs. 0.1.

$k_{11} =$ Rs. 1

$k_{12} =$ Rs. 0.98

$b = 800.$

$$Q_2^* = \sqrt{\frac{2DC_s}{k_{12}I}}$$

$$= \sqrt{\frac{2 \times 1600 \times 5}{0.98 \times 0.1}}$$

$$Q_2^* = 404 \text{ units}$$

Q_2^* does not satisfies $800 \leq Q_2$

$$\therefore Q_1^* = \sqrt{\frac{2DC_s}{K_{11} I}}$$

$$= \sqrt{\frac{2 \times 1600 \times 5}{1 \times 0.1}}$$

$$= \sqrt{160000}$$

$$Q_1^* = 400 \text{ units}.$$

$\therefore Q_1^*$ satisfies $0 \leq Q_1 < 800$.

$$T_c(Q_1^*) = \frac{D}{Q_1^*} C_s + DK_{11} + \frac{1}{2} Q_1^* K_{11} I.$$

$$= \frac{1600}{400} \times 5 + 1600 \times 1 + \frac{1}{2} \times 400 \times 1 \times 0.1$$

$$= 20 + 1600 + 20.$$

$$= \text{Rs. } 1640.$$

$$T_c(b) = \frac{D}{b} C_s + DK_{12} + \frac{1}{2} b K_{12} I.$$

$$= \frac{1600}{800} \times 5 + 1600 \times 0.98 + \frac{1}{2} \times 800 \times 0.98 \times 0.1$$

$$= 10 + 1568 + 39.2$$

$$= \text{Rs. } 1617.$$

$$T_c(b) < T_c(Q_1^*).$$

$\therefore$ optimum order quantity $Q = b$.

$Q = 800$ units.

3 The annual demand of a product is 10000 units. Each unit cost Rs 100, If orders placed in quantities below 200 units but for orders of 200 or above, the price is Rs. 95. The annual inventory holding cost is 10% of the value of the item and the ordering cost is Rs. 5 per order. Find the economic lot size?

Given $D = 10000$ units / year.

$C_s = Rs\ 5$.

$I = 10\% = Rs.\ 0.1$.

Quantity	Unit cost
$0 \le Q_1 < 200$	100.
$200 \le Q_2$	95

$K_{11} = Rs\ 100$

$K_{12} = Rs.\ 95$

$b = 200$.

$$Q_2^* = \sqrt{\frac{2DC_s}{K_{12}I}}$$

$$= \sqrt{\frac{2 \times 10000 \times 5}{95 \times 0.1}}$$

$$= 103 \text{ units}.$$

Q_2^* does not satisfies $200 \le Q_2$.

$$\therefore Q_1^* = \sqrt{\frac{2DC_s}{K_{11}I}}$$

$$= \sqrt{\frac{2 \times 10000 \times 5}{100 \times 0.1}}$$

$$= \sqrt{\frac{2 \times 10000 \times 5}{10}} = 100 \text{ units},$$ Q_1^* satisfies $0 \le Q_1 < 200$

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$$T_c(Q_1^*) = \frac{D}{Q_1^*} C_s + D K_{11} + \frac{1}{2} Q_1^* K_{11} I$$

$$= \frac{10000}{100} \times 5 + 10000 \times 100 + \frac{1}{2} \times 100 \times 100 \times 0.1$$

$$= 500 + 1000000 + 500$$

$$= Rs.\ 10,01,000$$

$$T_c(b) = \frac{D}{b} C_s + D K_{12} + \frac{1}{2} b K_{12} I$$

$$= \frac{10000}{200} \times 5 + 10000 \times 95 + \frac{1}{2} \times 200 \times 95 \times 0.1$$

$$= 250 + 950000 + 950$$

$$= Rs.\ 9,51,200$$

$$T_c(b) < T_c(Q_1^*)$$

$\therefore$ optimum order quantity $Q = b$

$Q = 200$ units.

4. Find the optimum order quantity for a product for which the price breaks as follows:

Quantity	Unit cost
$0 \le Q_1 < 500$	10
$500 \le Q_2 < 750$	9.25
$750 \le Q_3$	8.75

The monthly demand for the product is 200 units. The cost of storage is 2% of the unit cost and the cost of ordering is Rs. 350.

Given $D = 200$ units / month

$C_s = Rs.\ 350$

$I = 2\% = Rs.\ 0.02$

$K_{11} = Rs.\ 10$, $K_{12} = Rs.\ 9.25$, $K_{13} = Rs.\ 8.75$

$$Q_3^* = \sqrt{\frac{2DC_s}{k_{13} I}}$$

$$= \sqrt{\frac{2 \times 200 \times 350}{8.75 \times 0.02}}$$

$$= \sqrt{\frac{1,40,000}{0.175}}$$

$$= 894 \text{ units}$$

Q_3^* satisfies $750 \leq Q_3$.

$\therefore$ optimum order quantity $Q = Q_3^*$

$Q = 750$ units.

5 Find the optimum order quantity for a product for which the price breaks are as follows.

Quantity	unit cost
$0 \leq Q_1 < 50$	10
$50 \leq Q_2 < 100$	9
$100 \leq Q_3$	8

The monthly demand is 200 units, the cost of storage is 25% of the cost and the ordering cost is Rs. 20 per order

Given, $D = 200$ units / month.

$C_s = Rs.\ 20$

$I = 25\% = Rs\ 0.25$

$k_{11} = Rs.\ 10$

$k_{12} = Rs\ 9$

$k_{13} = Rs.\ 8$

$b_1 = 50$

$b_2 = 100$.

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$$Q_3^* = \sqrt{\frac{2DC_s}{k_{13}\, i}}$$

$$= \sqrt{\frac{2 \times 200 \times 20}{8 \times 0.25}}$$

$$= 63 \text{ units}$$

Q_3^* does not satisfies $100 \le Q_3$

$$Q_2^* = \sqrt{\frac{2DC_s}{k_{12}\, i}}$$

$$= \sqrt{\frac{2 \times 200 \times 20}{9 \times 0.25}}$$

$$= 60 \text{ units}$$

Q_2^* satisfies $50 \le Q_2 < 100$.

$$T_c(Q_2^*) = \frac{D}{Q_2^*} C_s + D k_{12} + \frac{1}{2} Q_2^* k_{12}\, i$$

$$= \frac{200}{60} \times 20 + 200 \times 9 + \frac{1}{2} \times 60 \times 9 \times 0.25$$

$$= \text{Rs. } 1934$$

$$T_c(b_2) = \frac{D}{b_2} C_s + D k_{13} + \frac{1}{2} b_2 k_{13}\, i$$

$$= \frac{200}{100} \times 20 + 200 \times 8 + \frac{1}{2} \times 100 \times 8 \times 0.25$$

$$= \text{Rs. } 1740.$$

$$T_c(b_2) < T_c(Q_2^*)$$

$\therefore$ optimum order quantity $Q = b_2$

$= 100$ units.

6 Find the optimum order quantity :

Quantity	unit cost
$0 \le Q_1 < 100$	20
$100 \le Q_2 < 200$	18
$200 \le Q_3$	16

The monthly demand is 400 units. The storage cost is 20% of the unit cost, ordering cost is Rs. 25.

Given: $D = 400$ units / month

$I = 20\% = 0.2$

$C_s = Rs. 25$

$k_{11} = Rs. 20$

$k_{12} = Rs. 18$

$k_{13} = Rs. 16$

$b_1 = 100$

$b_2 = 200$

$$Q_3^* = \sqrt{\frac{2DC_s}{k_{13} I}}$$

$$= \sqrt{\frac{2 \times 400 \times 25}{16 \times 0.2}}$$

$= 79$ units

Q_3^* does not satisfies $200 \le Q_3$

$$Q_2^* = \sqrt{\frac{2DC_s}{k_{12} I}}$$

$$= \sqrt{\frac{2 \times 400 \times 25}{18 \times 0.2}}$$

$= 75$ units

Q_2^* doesnot satisfies $100 \le Q_2 < 200$.

$$Q_1^* = \sqrt{\frac{2DC_s}{k_{11} I}}$$

$$= \sqrt{\frac{2 \times 400 \times 25}{20 \times 0.2}}$$

$$= 71 \text{ units}$$

Q_1^* satisfies $0 \le Q_1 < 100$

$$T_C(Q_1^*) = \frac{D}{Q_1^*} C_s + D k_{11} + \frac{1}{2} Q_1^* k_{11} I$$

$$= \frac{400}{71} \times 25 + 400 \times 20 + \frac{1}{2} \times 71 \times 20 \times 0.2$$

$$= \text{Rs. } 8283$$

$$T_C(b_1) = \frac{D}{b_1} C_s + D k_{12} + \frac{1}{2} b_1 k_{12} I$$

$$= \frac{400}{100} \times 25 + 400 \times 18 + \frac{1}{2} \times 100 \times 18 \times 0.2$$

$$= \text{Rs. } 7480$$

$$T_C(b_2) = \frac{D}{b_2} C_s + D k_{13} + \frac{1}{2} b_2 k_{13} I$$

$$= \frac{400}{200} \times 25 + 400 \times 16 + \frac{1}{2} \times 200 \times 16 \times 0.2$$

$$= \text{Rs. } 6770.$$

$$T_C(b_2) < T_C(b_1) < T_C(Q_1^*)$$

$\therefore$ optimum order quantity $Q = b_2$

$Q = 200$ units.

Homework:

1. Find the optimum order quantity:

Annual demand = 3600 units.

Ordering cost = Rs. 50.

Cost of storage = 20% of the unit cost.

Quantity	Unit cost
$0 \le Q_1 < 100$	20
$100 \le Q_2$	18.

2. Find the optimum order quantity.

Quantity	Unit cost
up to 699	10
700 and above	9.25.

The average yearly replacement is 2400 pallet. The ~~carrying~~ carrying cost is 12% of the average inventory and ordering cost per order is 100.

3. Find the optimal order quantity for a product for which the price breaks are as follows:

Quantity	Unit cost
$0 \le Q_1 < 500$	100.
$500 \le Q_2 < 750$	92.50
$750 \le Q_3$	87.50

The monthly demand for the product is 200 units. The holding cost is 2% of the unit cost and the cost of ordering is Rs. 1000.

4. A manufacturer of engines is required to purchase 4800 casting per year. monthly holding cost = 2%. ordering cost is Rs. 750

Find optimum order quantity

Quantity	Unit cost
$0 \le Q_1 < 500$	150
$500 \le Q_2 < 750$	140
$750 \le Q_3$	132

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Queuing Theory.

1. Definition: Waiting line (or) Queue.

A group of items waiting to receive service including those receiving the service is known as a waiting line (or) Queue

2. Customer:

The person waiting in a queue or receiving the service is called customer

3. Server:

The person by whom he is serviced is called a server

4. Characteristics of queuing system:

The basic queuing process can be described by

(i). The input (or arrival pattern)

(ii). Service discipline (or queue discipline)

(iii) Service Mechanism (or service pattern)

(iv). capacity of the system.

Input:

The input describes the pattern in which the customers arrive for service. Since, the servicing units (customers) arrive in a random fashion therefore their arrival pattern can be described interms of probabilities. Here in this chapter we assume that they arrive according to a poisson process.

Service Discipline:

The service discipline refers to the manner in which the members in the queue are chosen for service.

For example:

(i). First Come First Served (FCFS) (First In First Out) (FIFO)

According to this discipline the customers are served in the order of their arrival

Ex: Cinema ticket window,

Railway ticket window.

(ii). Last come First served (LCFS) (or) Last In First Out (LIFO)

According to this discipline to which the last arrival in the system is served first.

Ex: Files on the manager's Desk.

(iii) Service In Random order (SIRO)

This rule according to which the arrivals are served randomly irrespective of the arrivals in the system.

(iv) Service on some priority

Some customers are served before others without considering their order of arrival.

Service Mechanism:

The service mechanism is concerned with service time and service facilities.

Service time is the time interval from the commencement of service to the completion of last.

(i) If there are infinite number of servers, then all the customers are served instantaneously on arrival and there will be no queue.

(ii) If there are finite number of servers, then the customers are served according to order.

Service facilities can be of the following type:

(i). Single queue - one server.

(ii) Single queue - several servers.

(iii) Several queues - one server.

(iv) Several queues - several servers

Capacity of the system:

The source from which customers are may be finite or infinite.

5. Deterministic queuing system:

A queuing system where in the customers arrive at regular intervals and the service time for each customer is known and constant is known as deterministic queuing system.

6. Probabilistic queuing system:

It is assumed that customers joining the queuing system arrive in a random manner and follows a poisson distribution.

Note:

1) The number of arrivals in non-overlapping intervals are statistically independent.

(i.e). The process has independent increments.

2) The probability of more than one arrival between time t and time $t+\Delta t$ is $O(\Delta t)$.

(i.e). The probability of two or more arrivals during the small time interval Δt is negligible.

Thus. $P_0(\Delta t) + P_1(\Delta t) + O(\Delta t) = 1$

3) The probability that an arrival occurs between time t and $t+\Delta t$ is equal to $\lambda \Delta t + O(\Delta t)$

(i.e). $P_1(\Delta t) = \lambda \Delta t + O(\Delta t)$.

7. Distribution of Arrivals:

$$P_n(t) = \frac{(\lambda t)^n}{n!} e^{-\lambda t}$$

8. Transient state:

A system is said to be in transient state when its operating characteristics are dependent on time.

9. Steady - State :

A system is said to be in steady state when its operating characteristics becomes independent of time.

Note :

In steady - state model,

$\lim_{t \to \infty} P_n(t) = P_n$, $\lim_{t \to \infty} P_n'(t) = 0$.

10. Classification of queuing model :

A queuing model is symbolically represented as

$(a/b/c) : (d/e)$.

a - Distribution of inter arrival time

b - Distribution of inter service time

c - Number of servers

d - Capacity of the system.

e - queue discipline (service discipline).

11. Model : I $(M/M/1) : (\infty/FCFS)$.

(i). $P_n = (1-\rho)\rho^n$ (or) $P_n = \rho^n . P_0$

(ii). Average number of customers in the system

$$L_s = E(n) = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu-\lambda}$$

(iii) Average number of customers in the queue

$$L_q = E(m) = \frac{\rho^2}{1-\rho} = \frac{\lambda^2}{\mu(\mu-\lambda)}$$

(iv). Average length of non - empty queue

$$E(m/m>0) = \frac{\mu}{\mu-\lambda}$$

(v) Average waiting time of a customer in the system.

$$W_s = E(V) = \frac{1}{\mu - \lambda}$$

(vi) Average waiting time of a customer in the queue.

$$W_q = E(W) = \frac{\lambda}{\mu(\mu-\lambda)}$$

(vii) Average waiting time of an arrival who has to wait

$$E(W \mid W>0) = \frac{1}{\mu-\lambda}$$

(ix). Probability of no customers in the system.

$$P_0 = 1 - \rho, \quad \text{where } \rho = \lambda/\mu < 1$$

(x). Probability that there are more than n customers in the system is

$$P(>n) = \rho^{n+1}$$

(xi). Probability that there are more than n customers in the queue is

$$P(>n+1) = \rho^{n+2}.$$

1. A TV repairman finds that the time spent on his jobs as an exponential distribution with mean 30 minutes, if he repairs sets in the order in which they came in and if the arrival of sets is approximately poisson with an average rate of 10 per 8 hrs day.

(i). What is repairman's expected Idle time each day?

(ii) How many jobs are ahead of the average set just brought in?

$$\lambda = 10 \text{ sets / day}$$

$$\mu = 16 \text{ sets / day}$$

$$\rho = \frac{\lambda}{\mu} = \frac{10}{16} = \frac{5}{8} = 0.625$$

(i). The probability that the repairman to be idle is

$P_0 = 1 - \rho.$

$P_0 = 1 - \frac{5}{8} = \frac{3}{8} = 0.375$

Expected Idle time per day $= 8 \times 0.375 = 3$ hrs.

(or) $8 \times \frac{3}{8} = 3$ hrs

(ii) Average no of sets in the system.

$$L_s = \frac{\rho}{1-\rho}.$$

$$= \frac{0.625}{1-0.625} = \frac{0.625}{0.375} = 1.667$$

2. In a Supermarket the arrival range of customer rate of 5 every 30 minutes. The average time it takes to list and calculate the customer purchase at the cash ~~desk~~ desk is 4.5 minutes and this time is exponential distributed.

(i). How long will the customer expect to wait for service at the case desk.

(ii) what is the chance that the queue length will exceed 5.

(iii). what is the probability that cashier is working

Soln:

$$\lambda = \frac{5}{30} \text{ mins} = \frac{1}{6} \text{ mins}.$$

$$\mu = \frac{1}{4\frac{1}{2}} = \frac{1}{9/2} = \frac{2}{9} \text{ mins}$$

$$\rho = \lambda/\mu.$$

$$= \frac{1/6}{2/9} = \frac{9}{12} = \frac{3}{4} = 0.75 < 1$$

(i) Average waiting time of customer for the cash desk

$$W_q = E(w) = \frac{\lambda}{\mu(\mu-\lambda)}$$

$$= \frac{1/6}{2/9\left(2/9 - 1/6\right)}$$

$$= \frac{0.166}{0.0132} = 13.5 \text{ mins}$$

(ii) The chance that the queue length will exceed five

$$P(>n+1) = \rho^{n+2}$$

$$= \rho^{5+2}$$

$$= (0.75)^7$$

$$= 0.132$$

(iii) Probability that the cashier is working =

1 − Probability of no customer in the system

$$= 1 - P_0$$

$$= 1 - (1-\rho)$$

$$= 1 - 1 + \rho = \rho = 0.75.$$

3 In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter arrival time follows an exponential distribution and the service time distribution is also exponential with an average 36 minutes calculate the following.

(i). The mean queue size (line length)

(ii). The probability that the queue size exceeds to

(iii) If the input of trains increases to an average 33 per day, what will be the change in (i) & (ii).

Soln:

$$\lambda = 30 \text{ trains / day.}$$

$$\mu = \frac{1}{36} \text{ mins} = \frac{60 \times 24}{36} \text{ / day} = 40 \text{ / day.}$$

$$\rho = \frac{\lambda}{\mu} = \frac{30}{40} = 0.75 < 1$$

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(i). Mean queue size.

$$L_q = \frac{\rho^2}{1-\rho} = \frac{(0.75)^2}{1-0.75} = 2.25 \text{ trains.}$$

(ii). $P(\text{queue size} \geq 10) = \rho^{10} = (0.75)^{10}$

$= 0.0563$.

(iii). When the input increases to 33 trains per day.

$\lambda = 33/\text{day}$

$\mu = 40/\text{day}$.

$$\rho = \frac{\lambda}{\mu} = \frac{33}{40} = 0.825 < 1.$$

$\therefore$ Mean queue size :

$$L_q = \frac{\rho^2}{1-\rho} = \frac{(0.825)^2}{1-0.825} = \frac{0.6806}{0.175} = 3.89 \text{ trains}$$

$P(\text{queue size} \geq 10) = \rho^{10} = (0.825)^{10}$

$= 0.1461$

4. At a one man barber shop, customers arrive according to poisson distribution with a mean arrival rate of 5 per hour and his hair cutting time was exponentially distributed with an average hair cut taking 10 minutes. It is assumed that because of his excellent reputation, customers were always willing to wait. Calculate the following.

(i). Average number of customers in the shop and average number of customers waiting for a haircut.

(ii). The percent of time an arrival can walk right in without having to wait.

(iii). The percentage of customers who have to wait prior to getting into the barber's chair.

Soln

$$\lambda = 5/hr = \frac{5}{60} = \frac{1}{12}/min.$$

$$\mu = \frac{1}{10}/min.$$

$$\frac{\lambda}{\mu} = \frac{10}{12} = \frac{5}{6} = 0.83 < 1$$

(i) Average no. of customers in the shop.

$$L_s = \frac{\rho}{1-\rho} = \frac{0.83}{1-0.83} = \frac{0.83}{0.17} = 4.882 = 5$$

(ii) Average no. of customers in the queue.

$$L_q = \frac{\rho^2}{1-\rho} = \frac{(0.83)^2}{1-0.83} = 4.052 = 4$$

(iii) $P(\text{queue size} \geq 1) = \rho' = \rho = 0.83.3$

The percent of customer who have to wait $= \rho \times 100$
$= 0.83 \times 100$
$= 83.3\%$.

The percent of time an arrival can wait right in with out having to wait $= 100 - 83.3$
$= 16.7\%$.

5. A drive in bank window has a mean service time of 2 minutes, while the customer arrive at a rate of 20 per hour. Assuming that these represent rates with a poisson distribution determine

(i) The proportion the teller will be Idle
(ii) How long a customer will wait before reaching the server
(iii) what fraction of customer will have to wait in line
(iv) The probability that a customer has to wait

Soln:

$$\lambda = 20/hr \Rightarrow \frac{20}{60} \cdot \frac{1}{3}/min.$$

$$\mu = \frac{1}{2} \text{ mins}$$

$$\rho = \lambda/\mu = \frac{1/3}{1/2} = 2/3 = 0.667 < 1$$

(i). probability of no. of customers in the system

$$P_0 = 1 - \rho$$

$$= 1 - 0.667 = 0.333.$$

(ii). A customer will wait before reaching the server

$$W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{1/3}{1/2(1/2 - 1/3)} = 4 \text{ mins}.$$

(iii). The fraction of customer will have to wait in line

$$L_q = \frac{\rho^2}{1-\rho} = \frac{(0.667)^2}{1-0.667} = 1.336$$

(iv). Probability that a customer has to wait

$$= 1 - P_0$$

$$= 1 - (1-\rho)$$

$$= \rho = 0.667.$$

6. Consider a self service store with one cashier. Assume poisson arrivals and exponential service time. Suppose that 9 customer arrive on the average every 5 minutes and the cashier can serve 10 in 5 minutes. Find,

(i) The average number of customers queuing for service

(ii). The probability of having more than 10 customers in the system.

(iii) The probability that a customer has to queue for more than 2 minutes

If the service can be speeded up to 12 in 5 minutes by using a different cash register. what will be the effect on the quantities (i), (ii), (iii)

Soln :

$$\lambda = \frac{9}{5} \text{ /mins.}$$

$$\mu = \frac{10}{5} \text{ /mins.}$$

$$\rho = \frac{\lambda}{\mu} = \frac{9/5}{10/5} = \frac{9}{10} < 1$$

(i) Average no. of customers in the queue

$$L_q = \frac{\rho^2}{1-\rho} = \frac{(9/10)^2}{1 - 9/10} = 8.1$$

(ii) The probability of having more than 10 customers in the system.

$$P(\geq n+1) = \rho^{n+1}$$

$$= (0.9)^{10+1} = (0.9)^{11} = 0.3134$$

(iii) The probability that the customer has to queue for more than 2 mins is :

$$P(W \geq 2) = \int_2^\infty (\mu - \lambda) \cdot e^{-(\mu-\lambda)t} \cdot dt$$

$$= (\mu - \lambda) \int_2^\infty e^{-(\mu-\lambda)t} \cdot dt$$

$$= (\mu-\lambda) \cdot \left. \frac{e^{-(\mu-\lambda)t}}{-(\mu-\lambda)} \right|_2^\infty$$

$$= \left. -e^{-(\mu-\lambda)t} \right|_2^\infty$$

$$= e^{-2(\mu-\lambda)}.$$

$$= e^{-2\left(\frac{10}{5} - \frac{9}{5}\right)}$$

$$= e^{-2(1/5)}$$

$$= e^{-2/5} = e^{-0.4} = 0.67.$$

Also,

$$\lambda = 9/5 \text{ /min.}$$

$$\mu = 12/5 \text{ min.}$$

$$\rho = \frac{\lambda}{\mu} = \frac{9/5}{12/5} = \frac{9}{12} < 1$$

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(i). Average no. of customers in the queue.

$$L_q = \frac{\rho^2}{1-\rho} = \frac{(9/12)^2}{1-9/12} = 2.25$$

(ii). $P(\geq n+1) = \rho^{n+1}$

$$= (0.75)^{10+1}$$

$$= (0.75)^{11}$$

$$= 0.0422$$

(iii) The probability that the customer has to queue for more than 2 mins is.

$$P(w \geq 2) = \int_2^\infty (\mu-\lambda)\, e^{-(\mu-\lambda)t}\, dt$$

$$= (\mu-\lambda) \cdot \left. \frac{e^{-(\mu-\lambda)t}}{-(\mu-\lambda)} \right|_2^\infty$$

$$= \left. e^{-2(\mu-\lambda)} \right|_2^\infty$$

$$= e^{-2\left(\frac{12}{5}-\frac{9}{5}\right)}$$

$$= e^{-2(3/5)} = e^{-6/5} = 0.301$$

7. A Departmental store has a single during the rush hours customers arrive at a rate of 20 customer per hour the average no. of customer that can be processed by the cashier is 24 per hour. Assume that the conditions for use of single channel queuing model apply.

(i) what is the probability that the ~~customer~~ cashier is idle.

(ii). what is the average no of customer in the queuing system

(iii) what is the average time a customer spends in the system.

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(iv) what is the average no. of customer in the queue

(v) what is the average time a customer spends in the queue waiting for

Soln:

$\lambda = 20 / hr.$

$\mu = 24 / hr.$

$\rho = \frac{\lambda}{\mu} = \frac{22}{24} = 0.833.$

(i). The probability that the cashier is idle

$P_0 = 1 - \rho$

$= 1 - 0.833. = 0.167$

(ii). Average no. of customers in the system

$L_s = \frac{\rho}{1-\rho} = \frac{0.833}{1-0.833} = 4.988$

(iii) Average time a customer spends in the system.

$W_s = \frac{1}{\mu - \lambda} = \frac{1}{24-20} = \frac{1}{4} = 0.25$

(iv) Average no. of customers in the queue

$L_q = \frac{\rho^2}{1-\rho} = \frac{(0.833)^2}{1-0.833} = 4.156$

(v) Average time a customer spends in the queue

$W_q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{20}{24(24-20)} = \frac{20}{96} = 0.208$

Model II : (M/M/1) : (N/FCFS)

$$P_0 = \begin{cases} \dfrac{1-\rho}{1-\rho^{N+1}} & \rho \neq 1 \\ \dfrac{1}{N+1} & \rho = 1 \end{cases}$$

($\rho > 1$ is allowed)

$$P_n = \begin{cases} \dfrac{(1-\rho)\rho^n}{1-\rho^{N+1}} & \rho \neq 1, \quad 0 \leq n \leq N. \\ \dfrac{1}{N+1} & \rho = 1 \end{cases}$$

$$L_s = \sum_{n=0}^{N} n P_n = \frac{\rho\left[1-(N+1)\rho^N + N\rho^{N+1}\right]}{(1-\rho)(1-\rho^{N+1})} \cdot P_0 .$$

$$L_q = \sum_{n=1}^{N} (n-1) P_n = E(n) - \sum_{n=1}^{N} P_n .$$

$$L_q = \frac{\rho^2\left[1-N\rho^{N-1}+(N-1)\rho^N\right]}{(1-\rho)(1-\rho^{N+1})}$$

Model II : $(M/M/1):(N/FCFS)$

$$P_0 = \frac{1-\rho}{1-\rho^{N+1}}, \quad \rho = \frac{\lambda}{\mu}, \quad \rho > 1 \text{ is allowed}$$

$$P_n = \frac{1-\rho}{1-\rho^{N+1}} \cdot \rho^n. \quad \text{for } n = 0,1,2 \ldots N.$$

$$L_s = P_0 \sum_{n=0}^{N} n\rho^n. \quad (\text{or}) \; L_s = \frac{\rho\left[1-(N+1)\rho^N + N\rho^{N+1}\right]}{(1-\rho)(1-\rho^{N+1})}$$

$$L_q = L_s - \frac{\lambda}{\mu}. \quad (\text{or}) \; L_q = \frac{\rho^2\left[1-N\rho^{N+1}+(N-1)\rho^N\right]}{(1-\rho)(1-\rho^{N+1})}$$

$$W_s = \frac{L_s}{\lambda}.$$

$$W_q = \frac{L_q}{\lambda}$$

1. If for period of 2 hours in a day (8–10 AM) trains arrive at the yard every 20 minutes but the service time continues to remain 36 minutes, then calculate for this period

(i). The probability that the yard is empty.

(ii). average ~~queue length~~ number of trains in the system, assuming that capacity of the yard is 4 trains only.

<u>Soln</u>:

$$\lambda = \frac{1}{20} \text{ mins}$$

$$\mu = \frac{1}{36} \text{ mins}.$$

$$\rho = \frac{\lambda}{\mu} = \frac{36}{20} = 1.8.$$

$$N = 4.$$

(i). Prob that the yard is empty.

$$P_0 = \frac{\rho-1}{\rho^{N+1}-1} \quad (\because \rho > 1)$$

$$P_0 = \frac{1.8-1}{(1.8)^5-1} = \frac{0.8}{17.896} = 0.04.$$

(ii). Average no. of trains in the system.

$$L_s = P_0 \sum_{n=0}^{N} n.\rho^n$$

$$= P_0 \sum_{n=0}^{4} n.\rho^n$$

$$= P_0(\rho + 2\rho^2 + 3\rho^3 + 4\rho^4)$$

$$= 0.04(67.77)$$

$$= 2.9 \simeq 3 \text{ trains.}$$

2. Consider a single server queueing system with a poisson input, exponential service times. Suppose the mean arrive rate is 3 calling units per hour the expected service time is 0.25 hours and maximum permissible number calling units in the system is two. Derive the steady-state probability distribution of the number of calling units in the system and then calculate the expected number in the system.

soln:

$\lambda = 3$ units / hr.

$\mu = \frac{1}{0.25} = 4$ units / hr.

$\rho = \frac{\lambda}{\mu} = \frac{3}{4} < 1$, $1-\rho = 1 - \frac{3}{4} = 0.25$
$= 0.75$

$N = 2$

(i). The steady-state probability distribution of the number of calling units in the system is given by. $P_n = \frac{(1-\rho)}{1-\rho^{N+1}} . \rho^n.$

$$= \frac{(0.25)(0.75)^n}{1-(0.75)^3}.$$

$$P_n = (0.43)(0.75)^n.$$

$$P_0 = \frac{1-\rho}{1-\rho^{N+1}} = \frac{0.25}{1-(0.75)^3} = 0.43.$$

(ii). Expected number in the system

$$L_s = \sum_{n=0}^{N} n P_n$$

$$= \sum_{n=0}^{2} n(0.43)(0.75)^n$$

$$= (0.43)\sum_{n=0}^{2} n(0.75)^n$$

$$= (0.43)\left[1 \times 0.75 + 2(0.75)^2\right]$$

$$= 0.81.$$

3. Assume that the goods trains are coming in a yard at the rate of 30 trains per day and suppose that the inter-arrival times follow an exponential distribution. The service time for each train is assumed to be exponential with an average of 36 minutes. If the yard can admit 9 trains at a time (there being 10 lines, one of which is reserved for shunting purposes), calculate the probability that the yard is empty and find the average queue length.

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Soln :

$\lambda = 30 \text{ trains / day} = \frac{20}{60 \times 24} = \frac{1}{48} / \text{min}$

$\mu = \frac{1}{36} \text{ mins}$

$\rho = \frac{\lambda}{\mu} = \frac{1/48}{1/36} = \frac{3}{4} = 0.75 < 1 \quad , N = 9$

i) The probability that the yard is empty.

$$P_0 = \frac{1-\rho}{1-\rho^{N+1}}$$

$$= \frac{1-0.75}{1-(0.75)^{10}} = \frac{0.25}{0.90} = 0.2649$$

(ii) Average queue length.

$$L_q = P_0 \sum_{n=0}^{N} n\rho^n - \frac{\lambda}{\mu}$$

$$= \frac{1-0.75}{(1-0.75)^{10}} \sum_{n=0}^{9} n(0.75)^n - 0.75$$

$$= 0.28 \times (\rho + 2\rho^2 + 3\rho^3 + 4\rho^4 + 5\rho^5 + 6\rho^6 + 7\rho^7 + 8\rho^8 + 9\rho^9) - 0.75$$

$$= 0.28 \times 9.58 - 0.75$$

~~$\approx$ 3 trains~~

$$= 2.68 - 0.75$$

$$= 1.93 \simeq 2 \text{ trains}$$

—— X ——

4. Model III : $(M/M/c):(\infty/FCFS)$

$$P_n = \frac{\rho^n}{n!} P_0 \quad , \text{for } 1 \leq n \leq c \, ,$$

$$= \frac{1}{c^{n-c}\, c!} \rho^n P_0 \, , \quad n \geq c \, .$$

$$P_0 = \left[\sum_{n=0}^{c-1} \frac{1}{n!} \left(\frac{\lambda}{\mu}\right)^n + \frac{1}{c!} \left(\frac{\lambda}{\mu}\right)^c \frac{c\mu}{c\mu - \lambda} \right]^{-1} ,$$

(or)

$$P_0 = \left[\sum_{n=0}^{c-1} \frac{(c\rho)^n}{n!} + \frac{(c\rho)^c}{c!\,(1-\rho)} \right]^{-1} , \quad \rho = \frac{\lambda}{c\mu}$$

$$L_q = P_c \frac{\rho}{(1-\rho)^2} \qquad \text{where } P_c = \frac{(\lambda/\mu)^c \cdot P_0}{c!}$$

$$L_s = \frac{\lambda}{\mu} + L_q \, .$$

$$\text{prob}(w > 0) = \frac{P_c}{1-\rho} \, .$$

$$W_q = \frac{L_q}{\lambda} \, .$$

$$W_s = \frac{L_s}{\lambda} \, .$$

1. Four counters are being run on the frontier of a country to check the passports and necessary papers of the tourists. The tourists choose a counter at random. If the arrivals at the frontier is poisson at the rate λ and the service time is exponential with parameter $\lambda/2$, what is the steady-state average queue at each counter?

Soln :

$\lambda = \lambda.$

$\mu = \lambda/2$

$c = 4.$

$\rho = \dfrac{\lambda}{c\mu} = \dfrac{1}{2} \quad , \quad c\rho = 2.$

$$P_0 = \left[\sum_{n=0}^{c-1} \frac{(c\rho)^n}{n!} + \frac{(c\rho)^c}{c!(1-\rho)}\right]^{-1}$$

$$= \left[\sum_{n=0}^{3} \frac{2^n}{n!} + \frac{2^4}{4!(1-1/2)}\right]^{-1}$$

$$= \left[1 + 2 + \frac{2^2}{2!} + \frac{2^3}{3!} + \frac{16}{24} \times \frac{1}{1/2}\right]^{-1}$$

$$= \left[1 + 2 + \frac{4}{2} + \frac{8}{6} + \frac{32}{24}\right]^{-1}$$

$$= \left[1 + 2 + 2 + \frac{4}{3} + \frac{4}{3}\right]^{-1}$$

$$= \left[5 + \frac{4}{3} + \frac{4}{3}\right]^{-1} = \left[\frac{23}{3}\right]^{-1} = \frac{3}{23}.$$

(ii) Average queue length.

$$L_q = P_c \cdot \frac{\rho}{(1-\rho)^2} \qquad \because P_c = \frac{(c\rho)^c}{c!} \cdot P_0.$$

$$= \frac{(\lambda/\mu)^c}{c!} \cdot \frac{\rho}{(1-\rho)^2} \cdot P_0$$

$$= \frac{(c\rho)^c}{c!} \cdot \frac{\rho}{(1-\rho)^2} P_0$$

$$= \frac{2^4}{4!} \cdot \frac{1/2}{(1-1/2)^2} \cdot \frac{3}{23}.$$

$$= \frac{1.6}{24} \times \frac{1/2}{1/4} \times \frac{3}{23}$$

$$= \frac{\cancel{4}}{\cancel{16}_{2}} \times \frac{\cancel{4}^{2}}{\cancel{2}} \times \frac{\cancel{3}}{23}$$

$$L_q = \frac{4}{23}.$$

2. A telephone exchange has two long distance operator. The telephone company finds that during the peak load, long distance calls arrive in a poisson fashion at an average rate of 15 per hour. The length of service on these calls is approximately exponentially distributed with mean length of 5 minutes.

a) what is the probability that a subscriber will have to wait for his long distance call during the peak hours of the day?

b) If the subscribers will wait and are serviced in turn, what is the expected waiting time?

<u>soln</u>:

$$\lambda = 15 / hr. = \frac{15}{60} / min. = \frac{1}{4} min$$

$$\mu = \frac{1}{5} mins.$$

$$c = 2$$

$$\rho = \frac{\lambda}{c\mu} = \frac{5}{8}, \quad c\rho = 5/4.$$

$$P_0 = \left[\sum_{n=0}^{c-1} \frac{(c\rho)^n}{n!} + \frac{(c\rho)^c}{c!(1-\rho)} \right]^{-1}$$

$$P_0 = \left[\sum_{n=0}^{1} \frac{(5/4)^n}{n!} + \frac{(5/4)^2}{2!\,(1-5/8)}\right]^{-1}$$

$$= \left[1 + \frac{5}{4} + \frac{25/16}{2(3/8)}\right]^{-1}$$

$$= \left[1 + \frac{5}{4} + \frac{25}{\cancel{16}_{2} \times 2} \times \frac{\cancel{8}}{3}\right]^{-1}$$

$$= \left[1 + \frac{5}{4} + \frac{25}{12}\right]^{-1}$$

$$= \left[\frac{12+15+25}{12}\right]^{-1} = \left[\frac{52}{12}\right]^{-1} = \left[\frac{13}{3}\right]^{-1} = \frac{3}{13}.$$

a) $P(n \geq 2)$ = Probability that a subscriber will have to wait

(ie) $P(W > 0) = \dfrac{P_c}{1-\rho}.$

$$= \frac{(c\rho)^c}{c!\,(1-\rho)} \cdot P_0 \qquad \because P_c = \frac{(c\rho)^c}{c!} P_0.$$

$$= \frac{(5/4)^2}{2!\,(1-5/8)} \cdot \frac{3}{13}$$

$$= \frac{25}{\cancel{16}_{2}} \times \frac{1}{2} \times \frac{\cancel{8}}{\cancel{3}} \times \frac{\cancel{3}}{13}.$$

$$= \frac{25}{52} = 0.48.$$

(or)

$$p(n \geq 2) = \sum_{n=2}^{\infty} P_n$$

$$= 1 - P_0 - P_1$$

$$= 1 - \frac{3}{13} - \left(\frac{\lambda}{\mu} P_0\right)$$

$$= 1 - \frac{3}{13} - \left(\frac{5}{4} \times \frac{3}{13}\right)$$

$$= 1 - \frac{3}{13} - \frac{15}{52} = \frac{25}{52}$$

b) Expected waiting time in the queue

$$W_q = \frac{L_q}{\lambda}.$$

$$= P_c \frac{\rho}{(1-\rho)^2} \cdot \frac{1}{\lambda}.$$

$$= \frac{(c\rho)^c}{c!} \cdot \frac{\rho}{(1-\rho)^2} \cdot P_0 \cdot \frac{1}{\lambda}.$$

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$$= \frac{(5/4)^2}{2!} \cdot \frac{5/8}{(1-5/8)^2} \cdot \frac{3}{13} \times 4.$$

$$= \frac{25}{16 \times 2} \times \frac{5}{8} \times \frac{1}{(3/8)^2} \times \frac{3}{13} \times 4$$

$$= \frac{25}{\cancel{16}_2 \times \cancel{2}} \times \frac{5}{\cancel{8}} \times \frac{\cancel{64}^{8}}{\cancel{9}_3} \times \frac{\cancel{3}}{13} \times \cancel{4}.$$

$$L_q = \frac{125}{39} = 3.2 \text{ mins}$$

3. A supermarket has two girls ringing up sales at the counters. If the service time for each customer is exponential with mean 4 minutes, and if the people arrive in a poisson fashion at the rate of 10 per hour.

a) what is the probability of having to wait for service?

b) what is the expected percentage of idle time for each girl?

c) If a customer has to wait, what is the expected length of his waiting time?

Soln:

$$\lambda = 10 / hr. = \frac{10}{60} / min = \frac{1}{6} min.$$

$$\mu = \frac{1}{4} mins., \quad c = 2.$$

$$\rho = \frac{\lambda}{c\mu} = \frac{4}{2 \times 6} = \frac{1}{3}.$$

$$c\rho = 2/3.$$

$$P_0 = \left[\sum_{n=0}^{c-1} \frac{(c\rho)^n}{n!} + \frac{(c\rho)^c}{c!\,(1-\rho)} \right]^{-1}$$

$$= \left[\sum_{n=0}^{1} \frac{(2/3)^n}{n!} + \frac{(2/3)^2}{2!\,(1-1/3)} \right]^{-1}$$

$$= \left[1 + \frac{2}{3} + \frac{4}{9_3} \times \frac{1}{2} \times \frac{3}{2} \right]^{-1}$$

$$= \left[1 + \frac{2}{3} + \frac{1}{3} \right]^{-1}$$

$$= \left[1 + \frac{3}{3} \right]^{-1} = [1+1]^{-1} = 2^{-1} = \frac{1}{2}.$$

a) Prob. of having to wait for service

$$P(n \geq 2) = \sum_{n=2}^{\infty} P_n$$

$$= 1 - P_0 - P_1$$

$$= 1 - \frac{1}{2} - \frac{1}{3}.$$

$$= \frac{6-3-2}{6} = \frac{1}{6} = 0.167.$$

$$P_1 = \frac{\lambda}{\mu} P_0$$

$$= \frac{4}{6} \times \frac{1}{2} = \frac{1}{3}$$

b) Expected no. of idle girls $= 2.P_0 + 1.P_1$

$$= 2.\frac{1}{2} + 1.\frac{1}{3}$$

$$= 1 + \frac{1}{3}$$

$$= \frac{4}{3}.$$

Prob. of any girl being idle $= \dfrac{\text{Expected no. of idle girls}}{\text{Total no. of girls}}$

$$= \frac{4/3}{2} = \frac{4}{6} = \frac{2}{3}$$

$$= 0.67$$

$\therefore$ The expected percentage of idle time for each girl is 67 %.

c) Expected length of the customer's waiting time

$$= \frac{1}{c\mu - \lambda}$$

$$= \frac{1}{2\left(\frac{1}{4}\right) - \frac{1}{6}}$$

$$= \frac{1}{\frac{1}{2} - \frac{1}{6}}$$

$$= \frac{1}{\frac{3-1}{6}} = \frac{1}{2/6} = \frac{6}{2} = 3 \text{ mins}.$$