Sequencing Problem.

1. Definition: Sequencing:

The selection of an appropriate order for a series of jobs to be done on a finite number of service facilities in some pre-assigned order is called sequencing.

2. State the principal assumption of sequencing problem

(i). The processing times on different machines are exactly known and are independent of the order of the jobs in which they are to be processed.

(ii) The time taken by each job is changing over from one machine to another is negligible.

(iii) Each job, once started on a machine, is to be performed upto completion of that machine.

(iv). A job starts on the machine as soon as the job and machine both are idle and job is next to the machine and the ~~middle~~ machine is also next to the job.

(v) No machine may process more than one job simultaneously.

(vi) The order of completion of job has no significance (ie) no job is to be given priority, The order of completion of jobs is independent of sequence of jobs.

a. Processing Order

It refers to the order (sequence) in which given machines are required for completing the job.

b. Processing Time

It indicates the time required by a job on each machine.

c. Total Elapsed Time :

It is the time interval between starting the first job and completing the last job including the idle time (if any) in a particular order by the given set of machines.

d. Idle time :

It is the time for which a machine does not have a job to process.

(ie) Idle time from the end of job $(i-1)$ to the start of job i.

e. No Passing Rule

It refers, to the rule of maintaining the order in which jobs are to be processed on given machines.

Example

If n jobs are to be processed through 3 machines M_1, M_2 and M_3 in the order M_1, M_2 and M_3 then, this rule will mean that each job will go to Machine M_1, first then to M_2 and lastly to M_3.

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Processing n jobs through 2 machines:

Step: 1

List the jobs along with their processing times in a table as shown below.

Job number:		J_1	J_2	J_3	....	J_n
Processing Time	M_1:	t_{11}	t_{12}	t_{13}	....	t_{1n}
	M_2:	t_{21}	t_{22}	t_{23}	....	t_{2n}

Step: 2

Examine the rows for processing times on machines M_1 and M_2 and find the smallest processing time in each row

(i.e) Find out in (t_{1j}, t_{2j}) for all j.

Step: 3

If the smallest processing time is for the first machine M_1, then place the corresponding job in the first available position in the sequence If it is for the second machine, then place the corresponding job in the last available position in the sequence.

Step: 4

If there is a tie in selecting the minimum of all the processing times, then there may be three situations:

a) Minimum among all processing times is same for the machine.

(ie) $\min(t_{1j}, t_{2j}) = t_{1k} = t_{2r}$, then process the k^{th} job first and the r^{th} job last.

b) If the tie for the minimum occurs among processing times t_{1j} on machine M_1 only, then select arbitrarily the job to process first

c) If the tie for the minimum occurs among processing times t_{2j} on machine M_2, then select arbitrarily, the job to process last

step: 5

Cross off the jobs already assigned and repeat steps 1 through 4, placing the remaining jobs next to first or next to last, until all the jobs have been assigned.

step: 6

Calculate idle time for machines M_1 and M_2

a) Idle time for M_1 = Total Elapsed time − (Time when the last job in a sequence finishes on M_1)

b) Idle time for M_2 = Time at which the first job in a sequence finishes on M_1 + $\sum_{j=2}^{n}$ {(time when the j^{th} job in sequence starts on M_2 − time when the $(j-1)^{th}$ job in a sequence finishes on M_2)}

step: 7

The total Elapsed time to process all jobs through two machines is as under :

Total elapsed time = Time when the n^{th} job in a sequence finishes on machine M_2

$$= \sum_{j=1}^{n} t_{2j} + \sum_{j=1}^{n} I_{2j}$$

where,

t_{2j} = Time required for processing j^{th} job on machine

I_{2j} = Time for which machine M_2 remains idle after processing $(j-1)^{th}$ job and before starting work on j^{th} job.

9. Processing n jobs through k machines:

Step: 1

Find min t_{1j}, min t_{kj} and maximum of each of $t_{2j}, t_{3j} \ldots t_{k-1,j}$, for all $j = 1, 2 \ldots n$.

Step: 2

Check the following.

a) $\min t_{1j} \geq \max t_{ij}$, for $i = 2, 3 \ldots k-1$

(or) b) $\min t_{kj} \geq \max t_{ij}$, for $i = 2, 3 \ldots k-1$

Step: 3

If the inequalities of step 2 are not satisfied method fails. otherwise go to next step.

Step: 4.

Convert the k machines problem into a two machines problem by introduce two fictitious machines G and H, such that

$$t_{Gj} = t_{1j} + t_{2j} + \dots + t_{k-1,j}.$$

and $$t_{Hj} = t_{2j} + t_{3j} + \dots + t_{kj}.$$

Step 5

Determine the optimal sequence for the n jobs and 2 machines equivalent sequencing problem with the prescribed order GH in the same way as discussed earlier. The resulting sequence shall be optimum for the given problem.

Processing of n jobs through 2 machines:

Find the Total Elapsed Time and Idle Time.

1.

Job:	J_1	J_2	J_3	J_4	J_5	J_6
Machine A:	1	3	8	5	6	3
Machine B:	5	6	3	2	2	10

Solution:

(i). Sequence Table

J_1					

Smallest value 1 in Machine A

(ii). Next smallest value 2 in Machine B

also $2 \rightarrow 5$ ✓ J_4

$2 \rightarrow 6$ J_5

J_1				J_5	J_4

(iii) Next smallest value 3. in machine A & B

$3 \rightarrow 6$ & $3 \rightarrow 10$ in machine A

$3 \rightarrow 8$ in machine B

J_1	J_2	J_6	J_3	J_5	J_4

Total Elapsed Time Table:

Machine A					Machine B			
Job	T	In	out	Idle	T	In	out	Idle
J_1	1	0	1	–	5	1	6	1
J_2	3	1	4	–	6	6	12	–
J_6	3	4	7	–	10	12	22	–
J_3	8	7	15	–	3	22	25	–
J_5	6	15	21	–	2	25	27	–
J_4	5	21	26	–	2	27	29	–
								1

Total Elapsed Time = 29 hrs.

Idle time for machine A = 29 - 26 = 3 hrs.

Idle time for machine B = 1 hr.

2. Job : 1 2 3 4 5

Job	1	2	3	4	5
Machine A	5	1 (I)	9	3 (III)	10
Machine B	2 (II)	6	7	8	4 (IV)

Solution:

Sequence Table:

J_2	J_4	J_3	J_5	J_1

Total Elapsed time Table:

	Machine A				Machine B			
Job	T	In	out	Idle	T	In	out	Idle
J_2	1	0	1	-	6	1	7	1
J_4	3	1	4	-	8	7	15	-
J_3	9	4	13	-	7	15	22	-
J_5	10	13	23	-	4	23	27	1
J_1	5	23	28	-	2	28	30	1
								3

Total Elapsed time = 30 hrs.

Idle time for machine A = 30 - 28 = 2 hrs

Idle time for machine B = 3 hrs

3. Job : 1 2 3 4 5 6

Job	1	2	3	4	5	6
Machine A	5	9	4	7	8	6
Machine B	7	4	8	3	9	5

Solution:

Sequence Table.

J_3	J_1	J_5	J_6	J_2	J_4

Total Elapsed Time Table:

	Machine A				Machine B			
Job	T	In	Out	Idle	Time	In	Out	Idle
J_3	4	0	4	—	8	4	12	4
J_1	5	4	9	—	7	12	19	—
J_5	8	9	17	—	9	19	28	—
J_6	6	17	23	—	5	28	33	—
J_2	9	23	32	—	4	33	37	—
J_4	7	32	39	—	3	39	42	2
								6 hrs

Total Elapsed Time = 42 hrs.

Idle time for machine A = 42 - 39 = 3 hrs

Machine B = 6 hrs.

n jobs 3 machines:

1.

Job:	A	B	C	D	E	F	G
Machine M_1:	3	8	7	4	9	8	7
Machine M_2:	4	3	2	5	1	4	3
Machine M_3:	6	7	5	11	5	6	12

Solution:

Min $M_1 = 3$, Min $M_3 = 5$

Max. $M_2 = 5$.

Min $M_3 \geq$ Max M_2.

∴ This ~~Pro~~ 3 machines problem can be converted into 2 machines problem

Job :	A	B	C	D	E	F	G.
G_i :	3+4=7	8+3=11	7+2	4+5	9+1	8+4	7+3
H_i :	4+6	3+7	2+5	5+11	1+5	4+6	3+12.

∴ Job :	A	B	C	D	E	F	G.
	III			IV			VII
G_i :	7	11	9	9	10	12	10
H_i :	10	10	7	16	6	10	15.
		V	II		I	VI	

Sequence Table :

A	D	G	F	B	C	E

Total Elapsed Time Table :

	Machine M_1				Machine M_2				Machine M_3			
Job	T	In	out	Idle	T	In	out	Idle	T	In	out	Idle
A	3	0	3	–	4	3	7	3	6	7	13	7
D	4	3	7	–	5	7	12	–	11	13	24	–
G	7	7	14	–	3	14	17	2	12	24	36	–
F	8	14	22	–	4	22	26	5	6	36	42	–
B	8	22	30	–	3	30	33	4	7	42	49	–
C	7	30	37	–	2	37	39	4	5	49	54	–
E	9	37	46	–	1	46	47	7	5	54	59	–
								25				7

Total Elapsed Time = 59 hrs.

Idle time for Machine M_1 = 59 − 46 = 13 hrs

$M_2 = (59-47) + 25 = 12 + 25$

$= 37$ hrs.

$M_3 =$ 7 hrs.

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2 Job : 1 2 3 4 5 6

Job	1	2	3	4	5	6
Machine A :	12	10	9	14	7	9
B :	7	6	6	5	4	4
C :	6	5	6	4	2	4

Solution :

~~Sequence Table :~~

Min A_i = 7 , Min C_i = 2

Max B_i = 7.

Min $A_i \geq$ Max B_i .

∴ This Three machines problem can be converted into 2 machines problem.

Job	1	2	3	4	5	6
G_i :	19	16	15	19	11	13
H_i :	13	11	12	9	6	16
	VI	III	IV	II	I	V

Sequence Table :

J_6	J_1	J_3	J_2	J_4	J_5

Total Elapsed Time Table :

	Machine A				Machine B				Machine C			
Job	T	In	out	Idle	T	In	out	Idle	T	In	out	Idle
J_6	9	0	9	—	4	9	13	9	4	13	17	13
J_1	12	9	21	—	7	21	28	8	6	28	34	11
J_3	9	21	30	—	6	30	36	2	6	36	42	2
J_2	10	30	40	—	6	40	46	4	5	46	51	4
J_4	14	40	54	—	5	54	59	8	4	59	63	8
J_5	7	54	61	—	4	61	65	2	2	65	67	2
								33				40

Total Elapsed Time = 67 hrs

Idle time for Machine A = 67 − 61 = 6 hrs

machine B = (67 − 65) + 33

= 2 + 33 = 35 hrs.

Machine C = 40 hrs.

3.

Job	1	2	3	4	5
Machine A	8	10	6	7	11
Machine B	5	6	2	3	4
Machine C	4	9	8	6	5

Solution:

$Min\ A_i = 6, \quad Min\ C_i = 4$

$Max\ B_i = 6$

$Min\ A_i \geq max\ B_i$

This 3 machines problem can be converted in to 2 machines problem.

Job:	1	2	3	4	5
G_i:	13	16	8	10	15
H_i:	9	15	10	9	9

Sequence Table:

J_2	J_3	J_5	J_1	J_4

Total Elapsed Time table:

Job	Machine A				Machine B				Machine C			
	T	In	out	Idle	T	In	out	Idle	T	In	out	Idle
J_2	10	0	10	–	6	10	16	10	9	16	25	16
J_3	6	10	16	–	2	16	18	–	8	25	33	-
J_5	11	16	27	–	4	27	31	9	5	33	38	–
J_1	8	27	35	–	5	35	40	4	4	40	44	2
J_4	7	35	42	–	3	42	45	2	6	45	51	1
								25				19

Total Elapsed Time = 51 hrs.

Idle time for machine A = 51 − 42 = 9 hrs.

machine B = (51 − 45) + 25 = 6 + 25 = 31 hrs

machine C [illegible] hrs

Homework

1. Find the total Elapsed time.

1.

Job	A	B	C	D	E	F
M I :	1	4	6	3	5	2
M II :	3	6	8	8	1	5

Ans: 32 hrs

2.

Task :	A	B	C	D	E	F	G	H	I
Machine I :	2	5	4	9	6	8	7	5	4
Machine II :	6	8	7	4	3	9	3	8	11

Ans: 61 hrs

3.

Job :	1	2	3	4	5	6
Machine A :	8	3	7	2	5	1
B :	3	4	5	2	1	6
C :	8	7	6	9	10	9

Ans: 53 hrs

4.

Job :	1	2	3	4	5	6	7
Machine A :	7	8	6	6	7	8	5
B :	2	2	1	3	3	2	4
C :	6	5	4	4	2	1	5

Ans: 57 hrs

5.

Item	Machine A	B	C	D	E
I	9	7	4	5	11
II	8	8	6	7	12
III	7	6	7	8	10
IV	10	5	5	4	8

Ans: 66 hrs

6.

Job	M_1	M_2	M_3	M_4
J_1	25	15	14	24
J_2	22	12	20	22
J_3	23	13	16	25
J_4	26	10	13	29

Ans. 149 hrs.

n Jobs k machines:

1.

Item	Machine A	B	C	D
I	15	5	4	15
II	12	2	10	12
III	16	3	5	16
IV	17	3	4	17

Solution:

$\text{Min } A_i = 12$, $\text{Min } D_i = 12$

$\text{Max } B_i = 5$, $\text{Max } C_i = 10$

$\text{Min } A_i \geq \text{max } B_i$

This 4 machines problem can be converted in to 2 machines problem

Job	I	II	III	IV
G_i:	24	24	24	24
H_i:	24	24	24	24

Sequence Table:

I	II	III	IV

Total Elapsed Time Table:

Job	Machine A T	In	out	Idle	Machine B T	In	out	Idle	Machine C T	In	out	Idle	Machine D T	In	out	Idle
I	15	0	15	–	5	15	20	15	4	20	24	20	15	24	39	24
II	12	15	27	–	2	27	29	7	10	29	39	5	12	39	51	–
III	16	27	43	–	3	43	46	14	5	46	51	7	16	51	67	–
IV	17	43	60	–	3	60	63	14	4	63	67	12	17	67	84	–
								50				44				24

Total Elapsed Time = 84 hrs

Idle time for machine A = 84 − 60 = 24 hrs

machine B = (84 − 63) + 50 = 21 + 50 = 71 hrs

machine C = (84 − 67) + 44 = 17 + 44 = 61 hrs

machine D = 24 hrs.

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2.

Job	M_1	M_2	M_3	M_4	M_5	M_6
A	18	8	7	2	10	25
B	17	6	9	6	8	19
C	11	5	8	5	7	15
D	20.	4.	3	4.	8	12.

Solution:

Min $M_1 = 11$, Min $M_6 = 12$.

Max $M_2 = 8$, Max $M_3 = 9$, Max $M_4 = 6$

Max $M_5 = 10$.

Min $M_1 \geq$ max M_2

This 6 machines problem can be converted in to 2 machines problem.

Job:	A	B	C	D.
G_i:	45	46	36	39.
H_i:	52	48	40	31.

Sequence Table:

J_C	J_A	J_B	J_D

Job	Machine M_1				Machine M_2				Machine M_3				Machine M_4				Machine M_5				Machine M_6			
	T	In	out	Idle	T	In	out	Idle	T	In	out	Idle	T	In	out	Idle	T	In	out	Idle	T	In	out	Idle
C	11	0	11	-	5	11	16	11	8	16	24	16	5	24	29	24	7	29	36	29	15	36	51	36
[illegible]	18	11	29	-	8	29	37	13	7	37	44	13	2	44	46	15	10	46	56	10	25	56	81	7
[illegible]	17	29	46	-	6	46	52	9	9	52	61	8	6	61	67	15	8	67	75	11	19	81	100	-
[illegible]	20	46	66	-	4	66	70	14	3	70	73	9	4	73	77	6	8	77	85	2	12	100	112	-
								47				46				60				52				43

Total Elapsed Time = 112 hrs

Idle time for machine M_1 = $112 - 66 = 46$ hrs

machine M_2 = $(112 - 70) + 47 = 42 + 47 = 89$ hrs.

machine M_3 = $(112 - 73) + 46 = 39 + 46 = 85$ hrs.

machine M_4 = $(112 - 77) + 60 = 35 + 60 = 95$ hrs

machine M_5 = $(112 - 85) + 52 = 27 + 52 = 79$ hrs

machine M_6 = 43 hrs

Inventory Problems:

1. Definition: Inventory:

An inventory can be defined as a stock of goods, which is held for the purpose of future production or sales.

The stock of goods may be kept in the following forms:

(i). Raw material (ii) partly finished items
(iii). Finished goods (iv) spare parts . etc.

The objective of an inventory problem is to minimize total cost or to maximize the profit.

2. Reasons for carrying inventories:

(i). It helps in smooth and efficient running of business.

(ii) It provides adequate service to customers -

(iii) It reduces the possibility of duplicating of orders.

(iv). It improves the cashflow by timely shipment of customers orders.

(v). It helps in maintaining economy ~~of~~ by absorbing some of the fluctuations when the demand for an item fluctuates, or is seasonal.

(vi) It helps in minimizing the loss due to deterioration obsolence, damage or pilferage etc.

(vii). It acts as a buffer stock when raw materials are received late and shop rejections are too many.

(viii) Takes advantages of price discounts by bulk purchasing

(IX). It reduces the cost of product because of an added advantages of batching and long, ~~uniter~~ uninterrupted production runs.

(X). It improves the man power, equipment and facility utilization by better

3) Types of Inventory :

(I). Transportation inventory

(II). Buffer inventory

(III) Anticipation inventory

(IV). Decoupling inventory

(V). Lotsize inventory

4) Inventory costs :

Set up cost :

This is the cost associated with the setting up of machinery before starting production. Setup cost is generally assumed to be independent of the quantity ordered for or purchased.

Ordering Cost :

This is a cost associated with ordering of raw materials for production purposes. Advertisements consumption of stationery and postage, telephone charges, telegrams, rent for space used by the purchasing department, travelling expenditures incurred etc, constitute the ordering cost.

Purchase or Production Cost :

The cost of purchasing (or producing) a unit of an item is known as purchase (or production) cost. The purchase price will become important when quantity discounts are allowed for purchases above a certain quantity or when economics of sale suggest that the per unit production cost can be reduced by a larger production run.

Carrying or Holding cost:

The carrying cost is associated with carrying inventory. This cost generally includes the costs such as rent for space used for storage, interest on the money locked up, insurance for stored equipment, production, taxes, depreciation of equipment and furniture used etc

Shortage or Stockout Cost:

The penalty cost for running out of stock (i.e. when an item cannot be supplied on the customers demand) is known as shortage cost. This cost includes the loss of potential profit through sales of items and loss of good will, interms of permanent loss of customers and its associated lost profit in future sales.

Salvage cost or Selling price:

When the demand for certain commodity is affected by the quantity stocked, decision problem is based on a profit maximization criterion that includes the revenue from selling salvage value may be combined with the cost of storage and hence is generally neglected

5) Demand:

Demand is the number of items required per period which is not necessarily equal to the amount sold as some demand may be unfulfilled because of shortages or delays. The demand may be of two types.

Deterministic Demand :

If the number of items required (i.e. demands) in a subsequent period of time is known exact then such demands are called deterministic demands.

Non-Deterministic (or) Probabilistic Demand :

If the demands over a subsequent period of time is not known with certainly then such demands are called non-deterministic demands.

6. Lead Time :

The time gap between placing of an order and its actual arrival is the inventory is known as lead time.

7. Order Cycle :

The time period between placement of two successive orders is referred to as an order cycle.

8. Time Horizon :

The time period over which the inventory level will be controlled is called the time horizon. This horizon may be finite or infinite depending upon the nature of the demand for the commodity.

9. Re order level :

The level between maximum and minimum ~~stock~~ stock at which purchasing activities must start for replenishment is known as reorder level.

10. Classifications of inventory models :

The inventory problem may be classified into two categories :

(i) Deterministic models :

These are the inventory models in which demand is assumed to be fixed for a subsequent period of time

(ii). Probabilistic models:

These are the inventory models in which the demand is assumed to be a random variable are called probabilistic (or) stochastic method.

11 Economic order quantity:

The inventory problems in which demand is assumed to be fixed and completely predetermined are usually referred to as the Economic order quantity.

Economic order quantity is that size of order which minimizes total annual costs of carrying inventory and cost of ordering.

12 Model I: Deterministic inventory problem with no shortages:

(or)

Purchase inventory without shortages.

(or)

Uniform rate of demand, infinite rate of production with no shortages.

The objective of the study of this problem is to determine an optimum order quantity such that the total inventory cost is minimized. We illustrate the problem under consideration using the following assumptions:

(i) Demand is known and uniform.

(ii). Let 'D' denote the total number of units purchased (or) produced (or) supplied per time period and Q denote the lot size in each production run.

(iii) Shortages are not allowed.

(iv). Production rate is infinite.

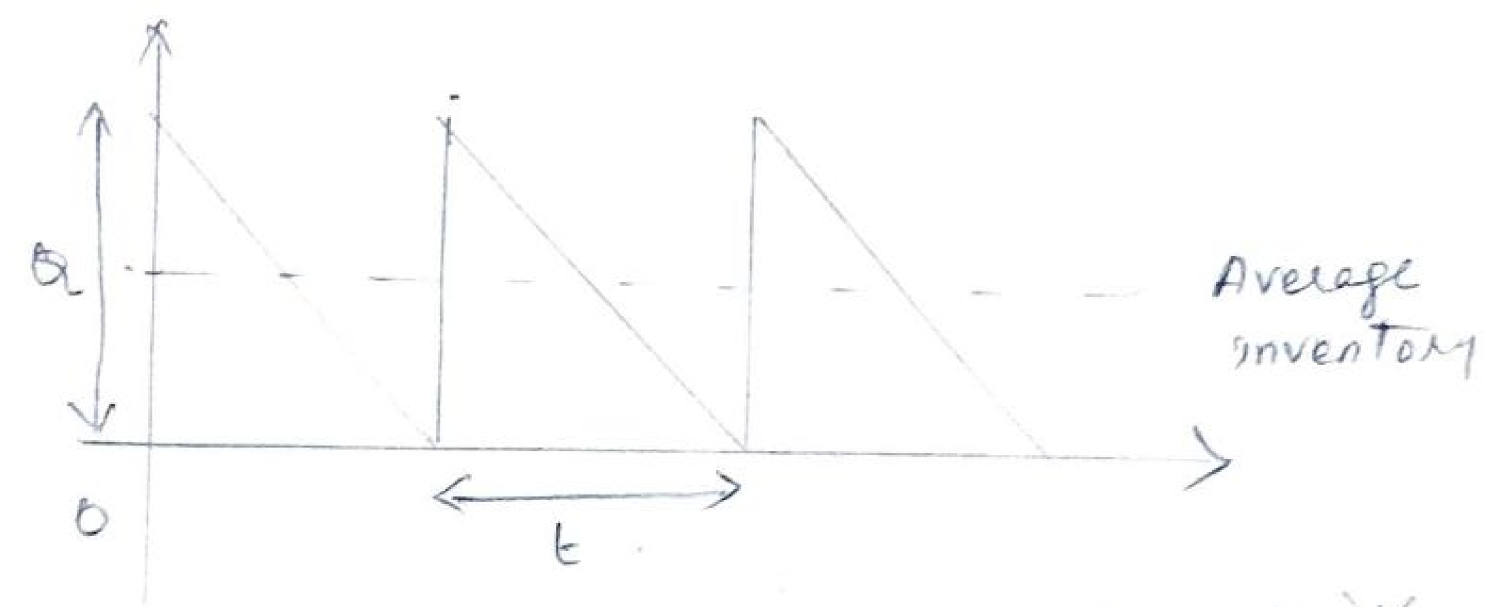
(v) Lead time is zero.

(vi). Setup cost (or ordering cost) per production run on procurement cost is C_s (or) C_o.

(vii) Holding cost (or) carrying cost is C_1 per unit in inventory for a unit time $C_1 = IC$.

Where C is the unit cost and I is the inventory carrying charge expressed of the values of the average inventory.

These fundamental situations can be shown on an inventory time diagram with Q on the vertical axis and time on the horizontal axis. The total time period is divided in to n parts.



If a production run is made at interval 't', a quantity $Q = Dt$ must be produced in each run. Since, the stock in small time 'dt' is $Dt\,dt$. The stock in time period 't' will be.

$$Q = Dt$$

$$\int_0^t Q\,dt = \int_0^t Dt\,dt$$

$$= \left[\frac{Dt^2}{2}\right]_0^t = \frac{1}{2} Dt \cdot t$$

$$= \frac{1}{2} Qt = \text{Area of inventory triangle}$$

Setup cost per production run = C_s.

Cost of holding inventory per production run = $\frac{1}{2} Dt^2 C_1$

Total Cost = $C_s + \frac{1}{2} Dt^2 C_1$

Average cost per unit time is

$$C(t) = \frac{C_s}{t} + \frac{1}{2} Dt\, C_1 \quad \text{——— (1)}$$

C will be minimum, if

$$\frac{dC(t)}{dt} = 0 \; ; \; \frac{d^2 C(t)}{dt^2} > 0$$

Diff. (1) w.r.to 't'

$$\frac{dC(t)}{dt} = -\frac{C_s}{t^2} + \frac{1}{2} DC_1$$

$$\therefore \; -\frac{C_s}{t^2} + \frac{1}{2} DC_1 = 0$$

$$\frac{C_s}{t^2} = \frac{1}{2} DC_1$$

$$t^2 = \frac{2C_s}{DC_1}$$

$$t = \sqrt{\frac{2C_s}{DC_1}}$$

Thus, $C(t)$ is minimum, for optimal time interval production run,

$$Q^* = Dt^*$$

$$= D\sqrt{\frac{2C_s}{DC_1}}$$

$$= \sqrt{\frac{2D^2 C_s}{DC_1}}$$

$$\boxed{Q = \sqrt{\frac{2DC_s}{C_1}}} \quad \text{(or)} \quad Q = \sqrt{\frac{2DC_s}{C_1}}$$

This is known as Economic lot size formula (or) R.H. Wilson's formula.

Note:

(i). Optimum number of orders placed per year,

$$n^* = D/Q = D\Big/\sqrt{\frac{2DC_S}{C_1}}$$

$$\boxed{n^* = \sqrt{\frac{DC_1}{2C_S}}}$$

(ii). Optimum length at time between orders (or) optimum order interval

$$t^* = \frac{Q}{D}$$

$$\boxed{t^* = \sqrt{\frac{2C_S}{DC_1}}}$$

(iii). Minimum total annual inventory cost

$$\boxed{T_C^* \text{ (or) } C_A = \sqrt{2DC_SC_1}}$$

Model : II

Problem of EOQ with finite replenishment.

(or)

Uniform rate of demand, finite rate of production with no shortages

(or)

Derive EOQ of production / Manufacturing model with no shortages.

In this case rate of production is none instantaneous but gradual. The following are the assumptions made:

(i). Demand D is uniform and known.

(ii) The replenishment made instantaneously.

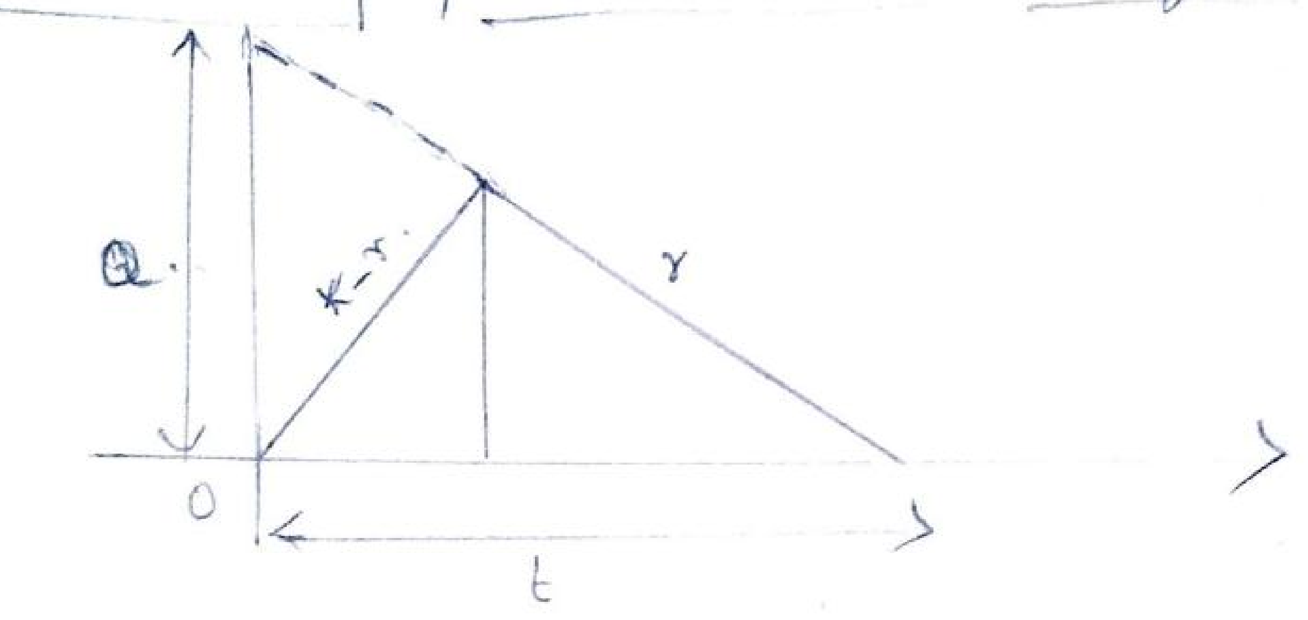
(iii) There is no shortages and no lead time.

(iv). Inventory carrying cost is C_1, Ordering cost is C_s.

(v) Let k be the production rate and r be the demand rate.

(vi). Let t be the length of the production run

The inventory production chart as follows:



Total production $Q = kt$

$t = Q/k$

Average inventory $= \frac{1}{2}(k-r)t$

$= \frac{1}{2}(k-r)Q/k$

$= \frac{Q}{2}(1-r/k)$

Total inventory carrying cost $= \frac{Q}{2}(1-r/k)C_1$

Total cost $=$ ordering cost $+$ carrying cost

$= nC_s + \frac{1}{2}.Q(1-r/k)C_1$

~~Diff w r to Q~~

Total minimum cost T_c (or) $C_A = \frac{r}{Q}C_s + \frac{1}{2}Q(1-\frac{r}{k})C_1$

Diff. w.r.to Q

$$\frac{dC_A(Q)}{dQ} = -\frac{r}{Q^2}C_s + \frac{1}{2}(1-r/k)C_1$$

$$\frac{d^2 C_n(Q)}{dQ^2} = \frac{2r}{Q^3} C_s > 0.$$

If C_n will be minimum.

$$\frac{d C_n(Q)}{dQ} = 0 \Rightarrow -\frac{r}{Q^2} C_s + \frac{1}{2}\left(1-\frac{r}{k}\right) C_1 = 0$$

$$\frac{r}{Q^2} C_s = \frac{1}{2}\left(\frac{k-r}{k}\right) C_1.$$

$$\frac{2rk\,C_s}{Q^2} = (k-r) C_1.$$

$$Q^2 = (k-r) C_1$$

$$Q^2 = \frac{2rk\,C_s}{(k-r) C_1}$$

$$Q = \sqrt{\frac{2rk\,C_s}{(k-r) C_1}}$$

ie. $$Q = \sqrt{\frac{2r C_s}{C_1} \cdot \frac{k}{k-r}}.$$

Total minimum cost $= \frac{r}{Q} C_s + \frac{1}{2} \cdot Q \left(1 - \frac{r}{k}\right) C_1$

$$= rC_s \cdot \sqrt{\frac{(k-r) C_1}{2rk\,C_s}} + \frac{1}{2}\sqrt{\frac{2rk\,C_s}{(k-r) C_1}} \left(\frac{k-r}{k}\right) C_1$$

$$= \sqrt{\frac{r^2 C_s^2 (k-r) C_1}{2 r k C_s}} + \sqrt{\frac{2 r k C_s}{4 (k-r) 2k} \cdot \frac{(k-r)^2}{k^2} \cdot C_1^2}$$

$$= \sqrt{\frac{r C_s (k-r) C_1}{2k}} + \sqrt{\frac{r C_s (k-r) C_1}{2k}}$$

$$= 2\sqrt{\frac{r C_s (k-r) C_1}{2k}} = \sqrt{\frac{4 r C_s (k-r) C_1}{2k}}$$

$$T_c = \sqrt{2 r C_s \cdot \left(\frac{k-r}{k}\right) C_1}$$

Note:

(i). Optimum number of order placed per year.

$$n^* = r/Q^*.$$

(ii) Optimum length at time between orders (or).

optimum ordering interval

$$t^* = Q^*/r.$$

(iii) Total Annual cost = (Cost per unit $\times r$) + $\sqrt{2 r C_s \cdot \left(\frac{k-r}{k}\right) C_1}$

Model : 2 : Problems :

$$Q = \sqrt{\frac{2DC_s}{C_1}}$$

$$T_c = \sqrt{2DC_sC_1}$$

$$N = D/Q \quad , \quad t = Q/D.$$

1. A manufacturer has to supply his customers with 600 units of his product per year. Shortages are not allowed and the shortage cost amounts to be Rs. 0.60 per unit per year. The setup cost per run is Rs. 80.00. Find the optimum run size and the minimum average yearly cost.

$D = 600$ units

$C_1 =$ Rs 0.60 / year

$C_s =$ Rs. 80.

$$Q = \sqrt{\frac{2DC_s}{C_1}}$$

$$= \sqrt{\frac{2 \times 600 \times 80}{0.60}}$$

$$= \sqrt{160000}$$

$Q = 400$ units

$$T_c = \sqrt{2DC_sC_1}$$

$$= \sqrt{2 \times 600 \times 80 \times 0.60}$$

$=$ Rs. 240

2) A Company has a steady-state demand of a product of 40 item per month. The purchase cost is Rs 6 per item and the cost of ordering and procuring the material is Rs. 15 per occassion. If the stock holding cost is 20% per annum. How frequently should the company replenish the stock?

$D = 40$ item / month

$C = Rs\ 6$

$C_s = Rs\ 15$

$I = 20\%$

$C_1 = C \times I = 6 \times \frac{20}{100} = Rs\ 1.2$

$$Q = \sqrt{\frac{2DC_s}{C_1}} =$$

$$= \sqrt{\frac{2 \times 40 \times 15}{1.2}} = \sqrt{\frac{1200}{1.2}} = 31.6$$

$Q = 32$ items

$t = Q/D$

$= 32/40$

$= 0.8$ Months

3) Using the following information, obtain EOQ and the total variable cost with the policy of ordering quantities of that size

Annual demand = 20,000

Set up cost = Rs 150 per order

Inventory carrying cost is 24% of average inventory value

$D = 20000$ units / year

$C_s =$ Rs. 150.

$C_1 = 24\% =$ Rs. 0.24

$$Q = \sqrt{\frac{2DC_s}{C_1}}$$

$$= \sqrt{\frac{2 \times 20000 \times 150}{0.24}}$$

$= 5000$ units.

$$T_C = \sqrt{2DC_sC_1}$$

$$= \sqrt{2 \times 20000 \times 150 \times 0.24}$$

$=$ Rs. 1200

4. A manufacturer has to supply his customer with 24000 units of his product per year. This demand is fixed and known. Since the unit is used by the customer has no storage space for the units. The manufacturer must ship a day's supply each day. If the manufacturer fails to supply the required units he will lose the amount and probably his business. Hence, the cost of holding inventory amounts to 0.10 per unit per month and the setup cost per run is Rs. 350. Find the optimum lot size and the length of optimum production run?

$D = 24000$ units / year.

$C_1 =$ Rs. 0.10/ month $=$ Rs. 1.2 / year.

$C_s =$ Rs. 350.

$$Q = \sqrt{2DC_s/C_1} = \sqrt{\frac{2 \times 24000 \times 350}{1.2}} = 3742 \text{ units}$$

$t = Q/D = 0.16$ year.

$n = D/Q = 6.41$

5. The demand for a particular item is 18,000 units per year. The holding cost per unit is Rs. 1.20 per year and the cost of one procurement is Rs. 400. No shortages are allowed and the replacement rate is instantaneous. Determine :

1) Optimum order quantity
2) Number of orders per year
3) Time between orders
4) Total cost per year when the cost of one unit is Rs.

$D = 18000$ units / year.

$C_1 = $ Rs. 1.20 / year.

$C_S = $ Rs. 400.

1) $Q = \sqrt{\frac{2DC_S}{C_1}}$

$= \sqrt{\frac{2 \times 18000 \times 400}{1.20}}$

$Q = 3464$ units.

2) $N = D/Q$

$= 18000 / 3464$

$= 5.2$ order / year.

3) $t = Q/D$

$= \frac{3464}{18000}$

$= 0.192$ / year.

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4) $T_c = \sqrt{2DC_sC_1} + I \times D$

$= \sqrt{2 \times 18000 \times 400 \times 1.20} + 1 \times 18000$

$= 4157 + 18000$

$=$ Rs. 22157.

6. xyz Company buys in lots of 2000 units in which ~~is~~ only 3 months supply. The cost per ~~year~~ unit is Rs. 125 and the order cost is Rs. 250. The inventory carrying cost is Rs. 20% of unit value. How much money can be saved by using economic order quantity?

$D = 2000$ units / 3 month.

$= 8000$ / year.

$C =$ Rs. 125

$C_s =$ Rs 250

$I = 20\%$, $C_1 = C \times I = \cancel{125}^{5} \times \frac{\cancel{20}^{5}}{\cancel{100}_{4}} =$ Rs. 25

$=$

$Q = \sqrt{\frac{2DC_s}{C_1}}$

$= \sqrt{\frac{2 \times 8000 \times \cancel{250}^{10}}{\cancel{25}_{1}}}$

$= \sqrt{160000}$

$= 400$ units.

$T_c = \sqrt{2DC_sC_1}$

$= \sqrt{2 \times 8000 \times 250 \times 25}$

$=$ Rs. 10000

Total cost for the existing policy.

$$T_c = \frac{1}{2} \times \frac{D}{4} \times C_1 + 4\,C_s$$

$$= \frac{1}{2} \times \frac{8000}{4} \times 25 + 4 \times 250$$

$$= 25000 + 1000$$

$$= \text{Rs. } 26000.$$

The money saved by using EOQ $= 26000 - 10000$

$= \text{Rs. } 16000.$

7. An electrical appliance manufacturer wishes to know what the economic quantity should be for a plastic impeller when the following information is available.

The average daily requirement is 120 units and the company has 250 working days a year, so that the total yearly requirement is approximately 30000 units a year. The manufacturing cost is 50 paise per part. The sum of the annual rate of interest, insurance, taxes and so forth is 20%. of the unit cost and the cost of preparation is Rs. 50 per lot.

$D = 30000$ units / year.

$C = 50$ paise $= \text{Rs. } 0.50$

$I = 20\%.$

$C_s = \text{Rs. } 50$

$C_1 = C \times I = 0.50 \times \frac{20}{100} = 0.1$

$$Q = \sqrt{\frac{2DC_s}{C_1}}$$

$$= \sqrt{\frac{2 \times 30000 \times 50}{0.1}} = 5477 \text{ units.}$$

8. Find the EOQ for the following: Annual usage = 1000 piece

Cost per piece = Rs. 250

Ordering cost = Rs. 6 / order.

Expending cost = Rs. 4 / order.

Holding cost = ~~Rs. 1~~ 20% / unit.

D = 1000 piece / year.

C_1 = Rs 250.

~~C_1 = Rs. 1 / unit.~~ I = 20% = 0.2.

C_s = 6 + 4 = Rs. 10 / order.

$$Q = \sqrt{\frac{2DC_s}{C_1}}$$

$$= \sqrt{\frac{2 \times \overset{4}{\cancel{1000}} \times 10}{\underset{1}{\cancel{250}} \times 0.2}}$$

$$= \sqrt{\frac{80}{0.2}}$$

$$= \sqrt{\frac{800}{2}}$$

Q = 20 pieces.

Model : II   Problems :

$$Q = \sqrt{\frac{2rC_s}{C_1} \cdot \frac{k}{k-r}}.$$

$$T_c = \sqrt{2rC_sC_1 \cdot \frac{k-r}{k}}.$$

where K = Production rate

r = Demand rate.

1. An item is produced at the rate of 50 item per day. The demand occurs at the rate of 25 items per day. If the set up cost is Rs. 100 and the holding cost is Rs 0.01 per unit of item per day. Find the economic lot size for one run, assuming that the shortages are not permitted. Also find the time of cycle and minimum total cost for one run.

K = 50 items / day.

r = 25 items / day.

C_s = Rs. 100.

C_1 = Rs. 0.01 / day.

$$Q = \sqrt{\frac{2rC_s}{C_1} \cdot \frac{k}{k-r}}$$

$$= \sqrt{\frac{2 \times 25 \times 100}{0.01} \cdot \frac{50}{50-25}}$$

$$= \sqrt{\frac{5000}{0.01} \times \frac{50}{25}}.$$

$= \cancel{Rs.}$ 1000 units.

$$t = Q/r = \frac{1000}{25} = 40 \text{ days}.$$

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$$T_C = \sqrt{2 r C_s C_1 \cdot \frac{k-r}{k}}.$$

$$= \sqrt{2 \times 25 \times 100 \times 0.01 \times \frac{25}{50}}.$$

$= Rs.\ 5$.

Minimum Total cost for one run = 5×40 = Rs. 200.

2. A contractor has to supply 10000 bearings per day to an automobile manufacturer. He finds that, when he starts a production run, he can produce 25000 bearings per day. The cost of holding a bearing in stock for one year is 2 ~~paise~~ and the set up cost of a production run is Rs. 18. How frequently should production run be made?

$k = 25000$ / day.

$r = 10000$ / day.

$C_1 = 2$ paise/yr = ~~Rs. 0.02/yr~~ = $\frac{0.02}{365}$ / day.

$= .000055$ / day.

$C_S = Rs.\ 18$.

$$Q = \sqrt{\frac{2 r C_s}{C_1} \cdot \frac{k}{k-r}}.$$

$$= \sqrt{\frac{2 \times 10000 \times 18}{0.000055} \cdot \frac{25000}{25000-10000}}$$

$$= \sqrt{\frac{2 \times 10000 \times \cancel{18}\,6}{0.000055} \times \frac{\cancel{25000}\,5}{\cancel{15000}\,3}}.$$

$= 104447$ bearings

$$t = Q/r = \frac{104447}{10000} = 10.4 \text{ days}.$$

Length of the production cycle $= \frac{104447}{25000}$

$= 4.0$ days.

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3. A contractor has to supply 20000 units per day. He can produce 30000 units per day. The cost of holding a unit in stock is Rs. 3 per year and the setup cost per run is Rs. 50. How frequently and of what size, the production run be made!

$K = 30000$ units / day.

$r = 20000$ units / day.

$C_1 = Rs. 3$ / year $= 3/365$ / day

$= Rs. 0.0082$.

$C_s = Rs. 50$.

$$Q = \sqrt{\frac{2rC_s}{C_1} \cdot \frac{k}{k-r}}$$

$$= \sqrt{\frac{2 \times 20000 \times 50}{0.0082} \cdot \frac{30000}{30000-20000}}$$

$$= \sqrt{\quad} = 27050 \text{ units}.$$

$$t = Q/r = \frac{27050}{20000} = 1.35 \text{ days}.$$

4. The demand for an item in a company is 18000 units per year, and the company can produce the item at a rate of 3000 per month. The cost of one setup is Rs. 500 and the holding cost of 1 unit per month is 15 paise. Determine the optimum manufacturing quantity and the total cost per year assuming the cost of 1 unit as Rs. 2.

$k = 3000$ / month $= 36000$ units / year.

$r = 18000$ units / year.

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C_s : Rs. 500.

C_1 = 15 paise / month = 0.15 / m. . 1.8 / year

$$Q = \sqrt{\frac{2rC_s}{C_1} \cdot \frac{k}{k-r}}.$$

$$= \sqrt{\frac{2 \times 18000 \times 500}{1.8} \times \frac{36000}{36000 - 18000}}$$

$$= \sqrt{\frac{2 \times 18000 \times 500}{1.8} \times \frac{36000}{18000}} = 4472 \text{ units}.$$

$$T_C = \sqrt{2rC_sC_1 \cdot \frac{k-r}{k}} + 2 \times r$$

$$= \sqrt{2 \times 18000 \times 500 \times 1.8 \times \frac{18000}{\cancel{2}36000}} + 2 \times 18000$$

= Rs. 40025.

5 An item is produced at the rate of 128 units per day. The annual demand is 6400 units. The setup cost for each production run is Rs. 24 and inventory carrying cost is Rs. 3 per unit per year. There are 240 working days for production each year. Develop an inventory policy for this item.

k = 128 units / day = $\overset{128 \times 240}{30720}$ / year.

r = 6400 units / year.

C_s = Rs. 24.

C_1 = Rs. 3.

$$Q = \sqrt{\frac{2rC_s}{C_1} \cdot \frac{k}{k-r}}.$$

$$= \sqrt{\frac{2 \times 6400 \times \overset{8}{\cancel{24}}}{\cancel{3}} \cdot \frac{30720}{30720 - 6400}}$$

$$= \sqrt{16 \times 6400 \times \frac{30720}{24320}} = 360 \text{ units}$$

$t = Q/r = \frac{360}{6400} = 0.056$ year $= 0.056 \times 240$

$= 13.44$ days

$= 13$ days.

$n = r/Q = \frac{6400}{360} = 17.78$.

$$T_C = \sqrt{2 r C_s C_1 \cdot \frac{k-r}{k}}$$

$$= \sqrt{2 \times 6400 \times 24 \times 3 \times \frac{24320}{30720}}$$

$= Rs.\ 854$.