Unit - I : Linear Programming.

1. Definition: OR.

OR is a Scientific approach to problems Solving for executive management

2. Characteristics of OR:

The essential characteristics of OR are:

(i). Its system orientation
(ii). The use of interdisciplinary terms.
(iii). Application of scientific method.
(iv). Uncovering of new problems.
(v). Use of Computer
(vi). Quantitative solution
(vii). Human factors.

3. Scope of OR (or). Applications of OR:

OR is a problem solving and decision making Science some of the areas of management where OR techniques have been successfully applied are,

(i). National plants and budgets
(ii). Defences services and battle fields operations.
(iii). Government developments and public sector units
(iv). Industrial establishments and private sector units
(v). Research and development engineering divisions.
(vi). Business management and marketing
(vii). Education and Training.
(viii). Transportation and Communication
(ix). Home management and Personal budgeting.

4. Phases of OR (or) OR Approach (or) How OR works.

OR like all scientific research is based on scientific methodology which proceeds along the following steps:

(i). Formulae the problem
(ii). Construct a mathematical model
(iii) Acquire the input data
(iv). Derive the solution from the model.
(v). Validate the model
(vi). Establish control over the solution
(vii) Implement the final results.

5. Linear Programming:

Linear programming is a technique for determining an optimum schedule of interdependent activities in view of the available resources.

6. Components of LPP:

LPP consists of 3 components
(i). Decision variable (ii). The objective (iii) The constraints

7. Assumptions of LPP:

(i) Certainity (ii). Divisibility
(iii). Proportionality (iv) Additivity.

8. Mathematical Formulation of ~~a~~ LPP:

The procedure for mathematical formulation of a LPP ~~are~~ as follows:

Step: 1 Study the given situation to find the key decisions to be made

Step 2 Identify the variables involved and designate them by symbols x_j ($j = 1, 2, \dots$)

Step 3 State the feasible alternatives which generally are $x_j \geq 0$, for all j.

Step 4 Identify the constraints in the problem and express them as linear inequalities or equations LHS of which are linear functions of the decision variables.

Step 5 Identify the objective function and express it as a linear function of decision variable.

9. General LPP:

Let z be a linear function on R^n defined by,

a) $z = c_1 x_1 + c_2 x_2 + \dots + c_n x_n$.

which c_j's are constants. Let a_{ij} be an $m \times n$ real matrix and let $\{b_1, b_2 \dots b_m\}$ be a set of constants such that,

b) $$a_{11} x_1 + a_{12} x_2 + \dots + a_{1n} x_n \geq \text{ or } \leq \text{ or } = b_1$$
$$a_{21} x_1 + a_{22} x_2 + \dots + a_{2n} x_n \geq \text{ or } \leq \text{ or } = b_2$$
$$\vdots$$
$$a_{m1} x_1 + a_{m2} x_2 + \dots + a_{mn} x_n \geq \text{ or } \leq \text{ or } = b_m$$

and finally, let

c) $x_j \geq 0$, $j = 1, 2 \dots n$

The problem of determining an ntuple $(x_1, x_2 \dots x_n)$ which makes z a minimum (or maximum) and which satisfies (b) and (c) is called the general LPP.

10. Objective function:

The linear function $z = c_1 x_1 + c_2 x_2 + \dots + c_n x_n$ which is to be minimized (or maximized) is called the objective function of general LPP.

11. Constraints:

The inequalities (b) are called the constraints of the general LPP.

12. Non-negative restrictions:

The set of inequalities (c) is usually known as the set of non-negative restrictions of the general LPP.

13. Solution:

An n-tuple $(x_1, x_2 \dots x_n)$ of real numbers which satisfies the constraints of a general LPP is called a solution.

14. Feasible solution:

Any solution to a general LPP which also satisfies the non-negative restrictions of the ~~random~~ problem is called a feasible solution.

15. Optimum solution:

Any feasible solution which optimizes (minimizes (or maximizes)) the objective function of a general LPP is called a optimum solution.

16. slack variable:

Let the constraints of a general LPP be

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i \quad , \quad i = 1, 2 \dots k.$$

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Then, the non-negative variable x_{n+i} which satisfy

$$\sum_{j=1}^{n} a_{ij} x_j + x_{n+i} = b_i, \quad i = 1,2 \ldots k$$

are called slack variables.

17. Surplus variables

Let the constraints of a general LPP be

$$\sum_{j=1}^{n} a_{ij} x_j \geq b_i, \quad i = k+1, k+2 \ldots l.$$

Then, the non-negative variables x_{n+i}, which satisfy,

$$\sum_{j=1}^{n} a_{ij} x_j - x_{n+i} = b_i, \quad i = k+1, k+2 \ldots l.$$

are called surplus variable

18. Canonical Form:

The general formulation of LPP can always be put in the following form:

$$\text{maximize } z = c_1 x_1 + c_2 x_2 + \ldots + c_n x_n$$

Subject to the constraints;

$$a_{i1} x_1 + a_{i2} x_2 + \ldots + a_{in} x_n \leq b_i, \quad i = 1,2 \ldots m$$

$$x_1, x_2 \ldots x_n \geq 0.$$

by making use of some elementary transformations. This form of LPP is called the canonical form of LPP.

19. Standard form:

The general LPP in the form

$$\text{maximize } ~~\text{or minimize}~~ \; z = c_1 x_1 + c_2 x_2 + \ldots + c_n x_n$$

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Subject to the Constraints

$$a_{i1} x_1 + a_{i2} x_2 + \dots + a_{in} x_n = b_i, \quad i = 1, 2 \dots n.$$

$$x_1, x_2 \dots x_n \geq 0$$

is known as standard form.

20 Infinite number of solutions:

If one of the constant equation is parallel to the objective function, then we get infinite number of solutions.

21. Unbounded solution:

A solution which increase (or) decrease the value of the objective function indefinitely is called an unbounded solution.

22. Basic Solution:

For a set of m equations in n variables $(n \geq m)$, a solution obtained by setting $(n-m)$ variables equal to zero and solving for remaining m equations in m variables is called a basic solution provided the determinant of the coefficient of these m variables is non zero.

These variables whose value didn't appear in the solution are called non-basic variables and the remaining variables are called basic variables

23. Basic Feasible solution:

A basic solution which also satisfies the non-negative restriction is called basic feasible solution.

The basic feasible solutions are of two types.

Degenerate:

A basic feasible solution is called degenerate if atleast one basic variable ~~posses~~ process zero value

Non-Degenerate:

A basic feasible solution is called non-degenerate if all basic variables are non-zero and positive

24 Alternative optimum solution: (AFS).

If in the optimum table the coefficient of a non-basic variable in the z, equation is zero, then there exists an alternative optimum solution to the given problem.

In this case, there exists infinite number of optimum solutions to the given problems.

25 Graphical Method: (search Approach method)

LPP involving two decision variables can easily be solved by graphical method.

Procedure:

The major steps in the solution of a linear programming problem by graphical method are summarised as follows:

Step. 1

Identify the problem. the decision variables, the objective and the restrictions.

Step: 2

Setup the mathematical formulation of the problem

Step: 3

Plot a graph representing all the constraints of the problem and identify the feasible region. The feasible region is the intersection of all the regions represented by the constraints of the problem and is restricted to the first quadrant only.

Step: 4

The feasible region obtained in step 3 may be bounded or unbounded. compute the coordinates of all the corner points of the feasible region.

Step: 5

Find out the value of the objective function at each corner (solution) point determined in step 4.

Step: 6

Select the corner point the optimizes (maximizes or minimizes) the value of the objective function - the optimum solution.

Step: 7 Interpret the results

26. Simplex method:

The simplex method is an iterative procedure by which we can, once any basic feasible solution has been determined, obtain a maximum (optimum basic feasible solution in a finite number of steps.

Step : 1

Check whether the objective function of the given LPP is to be maximized or minimized. If it is to be minimized then we convert it into a problem of maximizing it by using

minimum $z = -$ Maximum $(-z)$.

Step : 2

Check whether all b_i ($i = 1, 2, \ldots m$) are non-negative. If any one b_i is negative, then multiply the corresponding inequation of the constraints by -1, so as to get all b_i ($i = 1, 2 \ldots m$) non-negative.

Step : 3

Convert all the inequations of the constraints in to equations by introducing slack variables in the constraints. Put the cost of these variables equal to zero.

Step : 4

Obtain an initial basic feasible solution to the problem in the form $x_B = B^{-1} b$ and put it in the first column of the simplex table.

Step : 5

Compute the net evaluations $z_j - c_j$ ($j = 1, 2 \ldots n$) using the relations $z_j - c_j = c_B y_j - c_j$, where $y_j = B^{-1} a_j$ and $a_j \in A$

Examine the sign of $z_j - c_j$.

(i) If all $z_j - c_j \geq 0$ then the ~~Initial~~ Initial basic feasible solution x_B is an optimum basic

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feasible solution.

(ii) If atleast one $(z_j - c_j) < 0$, proceed on to the next step.

Step: 6

If there are more than one negative $z_j - c_j$ then choose the most negative of them. Let it be $z_r - c_r$, for some $j = r$.

(i). If all $y_{ir} \leq 0$, $(i = 1, 2 \ldots m)$, then there is an unbounded solution to the given problem.

(ii). If atleast one $y_{ir} > 0$ $(i = 1, 2 \ldots m)$ then the corresponding vector y_r enters the basis y_B.

Step: 7 Compute the ratios $\left\{ \frac{x_{Bi}}{y_{ir}}, y_{ir} > 0, i = 1, 2 \ldots m \right\}$ and choose the minimum of them. Let the minimum of these ratios be $\frac{x_{Bk}}{y_{kr}}$. Then the vector y_k will leave the basis y_B. The common element y_{kr}, which is in the k^{th} row and the r^{th} column is known as the leading element (or pivotal element or key number)

Step: 8 Convert the leading element to unity by dividing its row by the leading element itself and all other elements in its column to zero by making use of the relations:

$$\hat{y}_{ij} = y_{ij} - \frac{y_{kj}}{y_{kr}} y_{ir}, \quad i = 1, 2 \ldots m+1, \; i \neq k$$

$$\text{and} \quad \hat{y}_{kj} = \frac{y_{kj}}{y_{kr}}, \quad j = 0, 1, 2 \ldots n.$$

Step: 9 Go to step 5 and repeat the computational procedure until either an optimum solution is obtained

Or there is an indication of an unbounded solution.

Graphical Method

1. Find the feasible solution:

maximize $z = 4x_1 + 3x_2$

Subject to $2x_1 + x_2 \le 1000$

$x_1 + x_2 \le 800$

$0 \le x_1 \le 400$

$0 \le x_2 \le 700$

Soln:

(i). Take, $2x_1 + x_2 \le 1000$

Let, $2x_1 + x_2 = 1000$

Sub, $x_1 = 0 \Rightarrow x_2 = 1000$.

$\therefore$ The point is $(0, 1000)$

Sub $x_2 = 0 \Rightarrow x_1 = 500$

$\therefore$ The point is $(500, 0)$.

(ii) Take, $x_1 + x_2 \le 800$.

Let, $x_1 + x_2 = 800$.

Sub, $x_1 = 0 \Rightarrow x_2 = 800$

$\therefore$ The point is $(0, 800)$

Sub $x_2 = 0 \Rightarrow x_1 = 800$

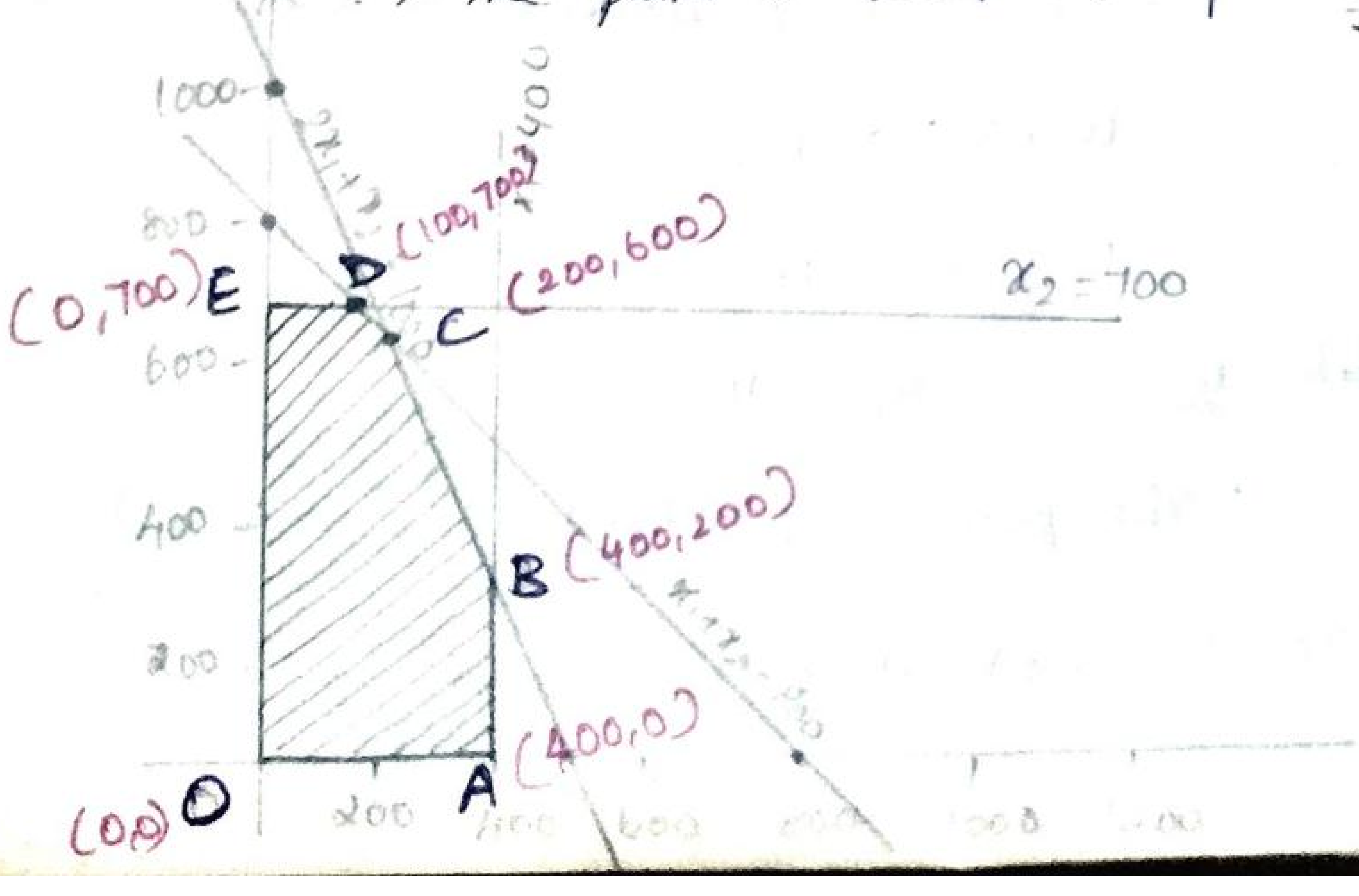
$\therefore$ The point is $(800, 0)$.

(iii) Take $0 \le x_1 \le 400$

$x_1 = 400$.

(iv). and $0 \le x_2 \le 700$

$x_2 = 700$



For B, $x_1 = 400$, $2x_1 + x_2 = 1000$ } solve

$x_2 = 200$

For C, solve $\begin{cases} 2x_1 + x_2 = 1000 \\ x_1 + x_2 = 800 \end{cases}$

$x_1 = 200$, $x_2 = 600$

For D, $x_1 + x_2 = 800$

$x_2 = 700$

$x_1 = 100$,

The feasible region is OABCDE.

Extreme points	(x_1, x_2)	$Z = 4x_1 + 3x_2$
O	(0, 0)	0
A	(400, 0)	1600
B	(400, 200)	2200
C	(200, 600)	2600 → maximum
D	(100, 700)	2500
E	(0, 700)	2100

∴ The optimum solution is

$x_1 = 200, \quad x_2 = 600.$

$\max Z = 2600.$

2. Find the minimum value of $Z = 7y_1 + 8y_2$

subject to the constraints

$3y_1 + y_2 \geq 8$

$y_1 + 3y_2 \geq 11.$

$y_1, y_2 \geq 0.$

Solution:

(i). Take, $3y_1 + y_2 \geq 8$.

$3y_1 + y_2 = 8$.

Sub $y_1 = 0 \Rightarrow y_2 = 8$.

∴ The point is $(0, 8)$

Sub $y_2 = 0 \Rightarrow y_1 = 8/3$

∴ The point is $(8/3, 0) \Rightarrow (2.6, 0)$

(ii). Take, $y_1 + 3y_2 \geq 11$.

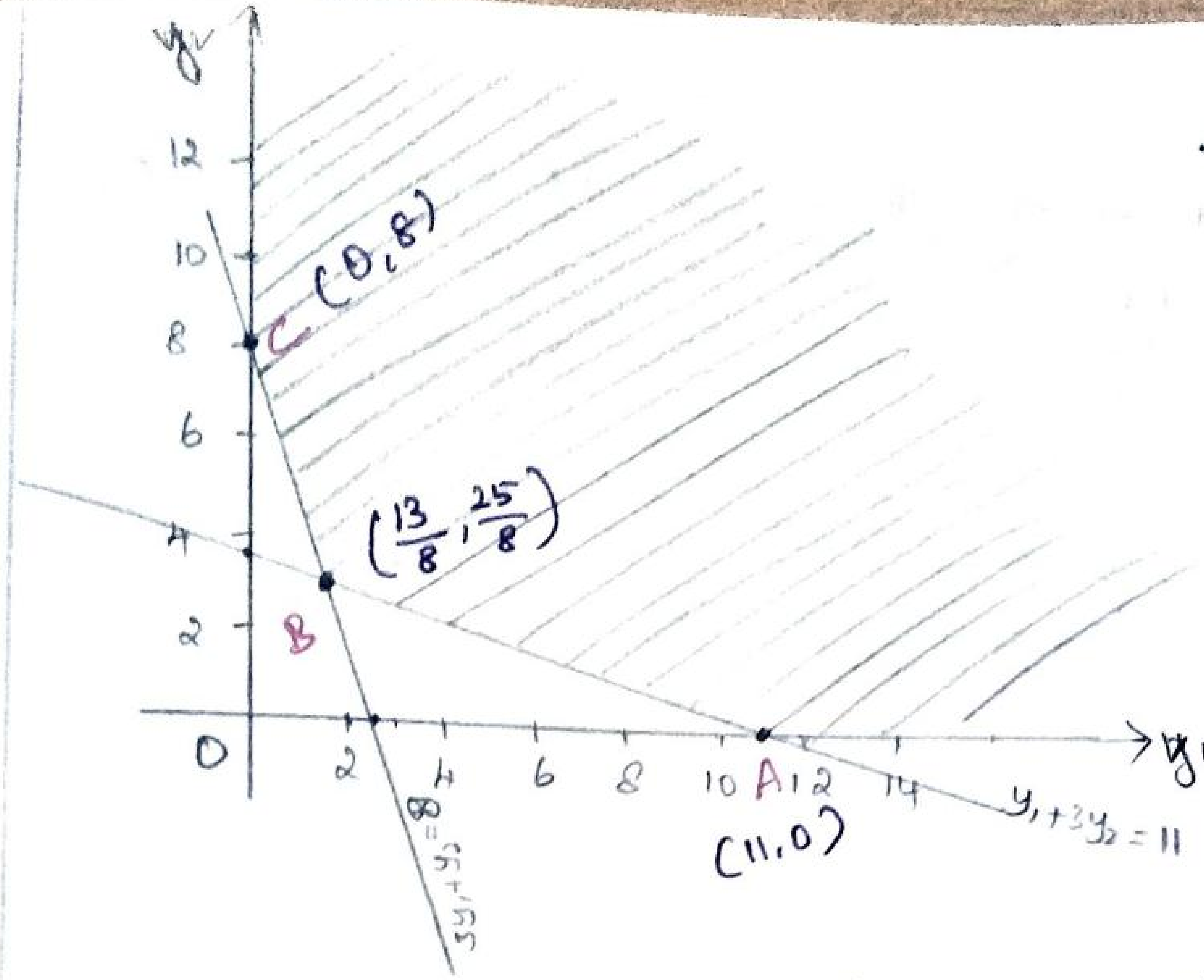
$y_1 + 3y_2 = 11$

Sub $y_1 = 0 \Rightarrow y_2 = 11/3$

∴ The point is $(0, 11/3) \Rightarrow (0, 3.6)$

Sub $y_2 = 0 \Rightarrow y_1 = 11.$

∴ The point is $(11, 0)$.



For B

$y_1 + 3y_2 = 11$ —①

$3y_1 + y_2 = 8$ —②

①×3 ⇒ $3y_1 + 9y_2 = 33$

② ⇒ $\underline{3y_1 + y_2 = 8}$

$8y_2 = 25$

$y_2 = 25/8$

$y_1 + 3 \times \frac{25}{8} = 11$

$y_1 + \frac{75}{8} = 11$

$y_1 = 11 - \frac{75}{8}$

$= \frac{88-75}{8}$

$y_1 = \frac{13}{8}$

The region is unbounded. But the objective function is minimum.

Extreme points	(y_1, y_2)	$Z = 7y_1 + 8y_2$
A	(11, 0)	77
B	$\left(\frac{13}{8}, \frac{25}{8}\right)$	36.4 ⟵ min.
C	(0, 8)	64

∴ The optimum solution is

$y_1 = \frac{13}{8}$, $y_2 = \frac{25}{8}$

$\min Z = 36.4$

3. Find the maximum value of $z = 3x_1 + 4x_2$ ~~sub~~

subject to $\quad x_1 - x_2 \geq -1 \quad \Rightarrow -x_1 + x_2 \leq 1$

$-x_1 + x_2 \leq 0$

$x_1, x_2 \geq 0$.

Soln:

(i). Take $\quad x_1 - x_2 \geq -1$

$-x_1 + x_2 \leq 1$

Let $\quad -x_1 + x_2 = 1$.

Sub $x_1 = 0 \Rightarrow x_2 = 1 \Rightarrow$ The point is $(0, 1)$

Sub $x_2 = 0 \Rightarrow x_1 = -1 \Rightarrow$ The point is $(-1, 0)$

(ii) Take, $\quad -x_1 + x_2 \leq 0$.

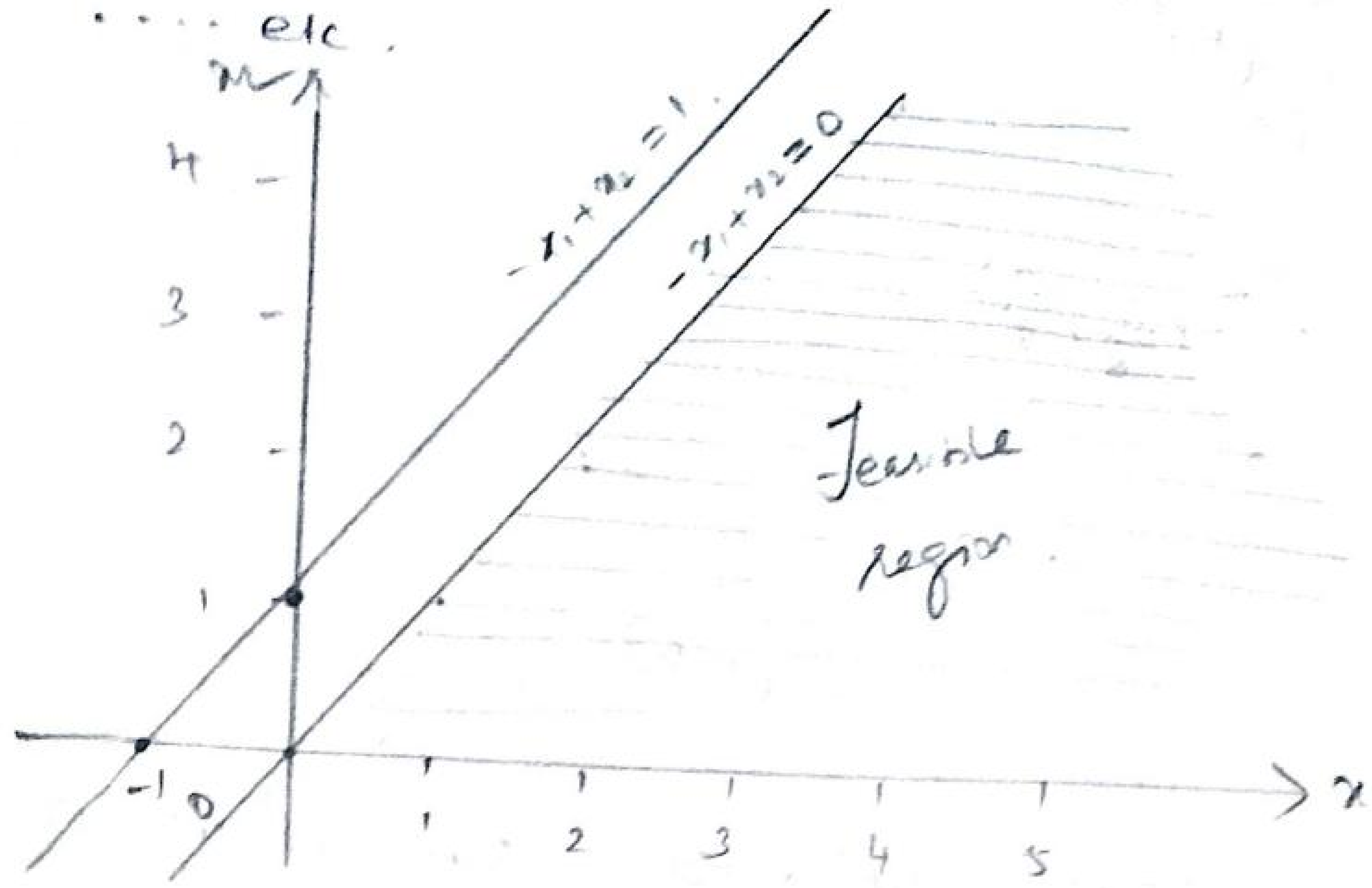
Let $\quad -x_1 + x_2 = 0$.

Sub $x_1 = 0 \Rightarrow x_2 = 0 \Rightarrow$ The point is $(0, 0)$.

Sub $x_1 = 1 \Rightarrow x_2 = 1 \Rightarrow$ The point is $(1,1)$

m^{ly}, $x_1 = 2 \Rightarrow x_2 = 2 \Rightarrow$ The point is $(2,2)$

.... etc.



The feasible region is unbounded.

But, the objective function is maximum.

$\therefore$ The given problem has an unbounded solution.

4. Find the maximum value of $z = -5x_2$

Subject to $x_1 + x_2 \le 1$

$-0.5x_1 - 5x_2 \le -10$

$x_1, x_2 \ge 0$

Soln:

(i) Take, $x_1 + x_2 \le 1$.

Let $x_1 + x_2 = 1$.

Sub $x_1 = 0 \Rightarrow x_2 = 1 \Rightarrow$ The point is $(0,1)$

Sub $x_2 = 0 \Rightarrow x_1 = 1 \Rightarrow$ The point is $(1,0)$

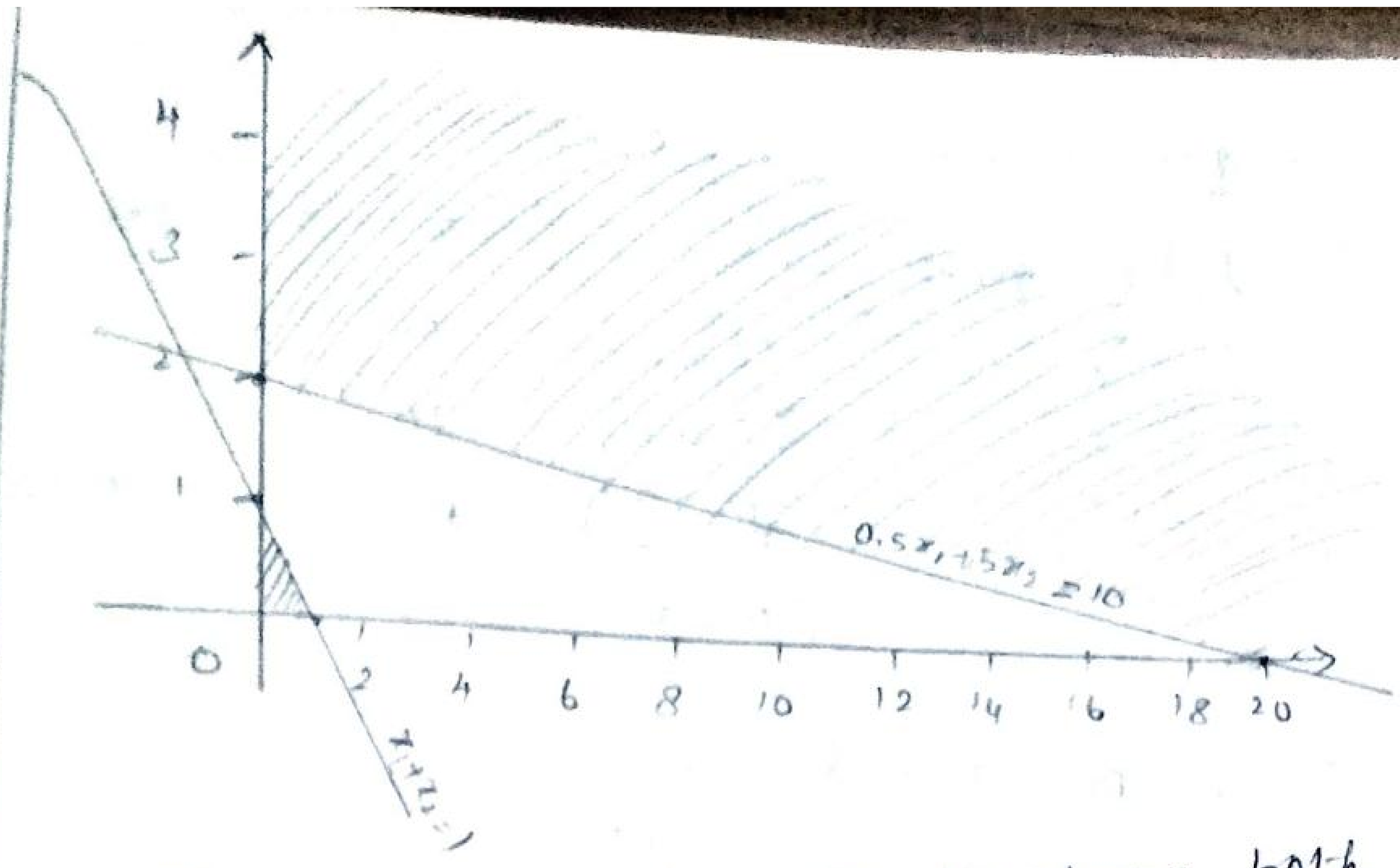
(ii) Take, $-0.5x_1 - 5x_2 \le -10$.

$0.5x_1 + 5x_2 \ge 10$.

Let $0.5x_1 + 5x_2 = 10$.

Sub $x_1 = 0 \Rightarrow x_2 = 2 \Rightarrow$ The point is $(0,2)$

Sub $x_2 = 0 \Rightarrow x_1 = 20 \Rightarrow$ The point is $(20,0)$



There is no set of points satisfying both the constraints. (i.e. No feasible region).

$\therefore$ There is no feasible solution.

5. Find the minimum value of $z = 5x_1 + 3x_2$

subject to $3x_1 + 5x_2 \leq 15$

$5x_1 + 2x_2 \leq 10$

$x_1, x_2 \geq 0$.

Soln:

(i). Take, $3x_1 + 5x_2 \leq 15$

let, $3x_1 + 5x_2 = 15$

Sub $x_1 = 0 \Rightarrow x_2 = 3 \Rightarrow$ The point is $(0, 3)$

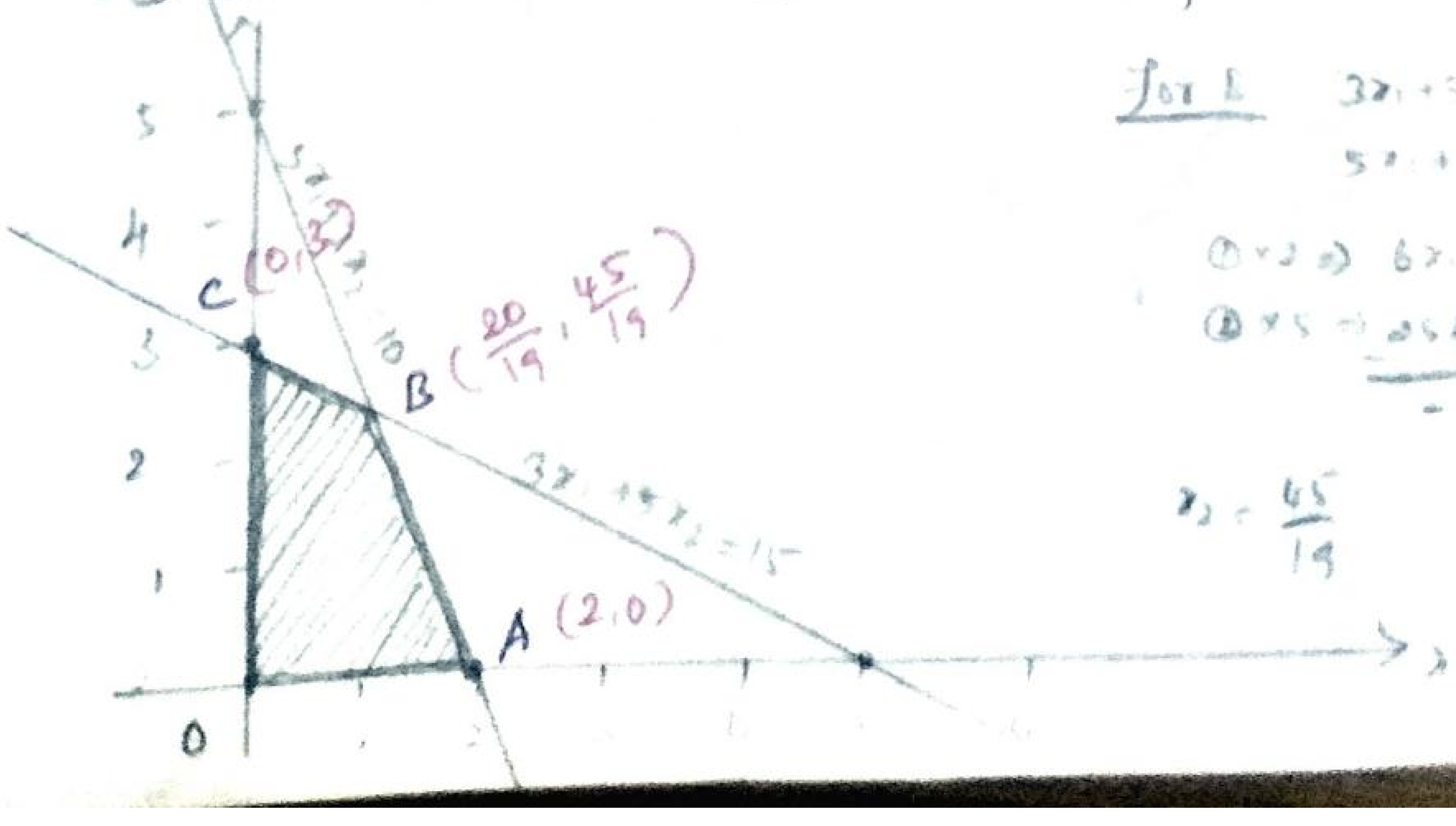
sub $x_2 = 0 \Rightarrow x_1 = 5 \Rightarrow$ The point is $(5, 0)$.

(ii). Take, $5x_1 + 2x_2 \leq 10$

let $5x_1 + 2x_2 = 10$

Sub $x_1 = 0 \Rightarrow x_2 = 5 \Rightarrow$ The point is $(0, 5)$

sub $x_2 = 0 \Rightarrow x_1 = 2 \Rightarrow$ The point is $(2, 0)$



For B $3x_1 + 5x_2 = 15$ — ①

$5x_1 + 2x_2 = 10$ — ②

① × 2 ⇒ $6x_1 + 10x_2 = 30$

② × 5 ⇒ $25x_1 + 10x_2 = 50$

$-19x_1 = -20$

$x_1 = \frac{20}{19}$

$x_2 = \frac{45}{19}$

The feasible region is OABC.

Extreme points	(x_1, x_2)	$z = 5x_1 + 3x_2$
O	$(0,0)$	0
A	$(2,0)$	10
B	$\left(\frac{20}{19}, \frac{45}{19}\right)$	12.7 ⟵ max.
C	$(0,3)$.	9

$\therefore$ The optimum soln is

$$x_1 = \frac{20}{19}, \quad x_2 = \frac{45}{19}.$$

max. $z = 12.7$.

6. Find the minimum of the function $z = 2x - y$

Subject to $x + y \leq 5$,

$x + 2y \geq 8$

$x, y \geq 0$.

Soln:

(i). Take $x + y \leq 5$

$x + y = 5$

Sub $x = 0 \Rightarrow y = 5 \Rightarrow$ The point is $(0,5)$

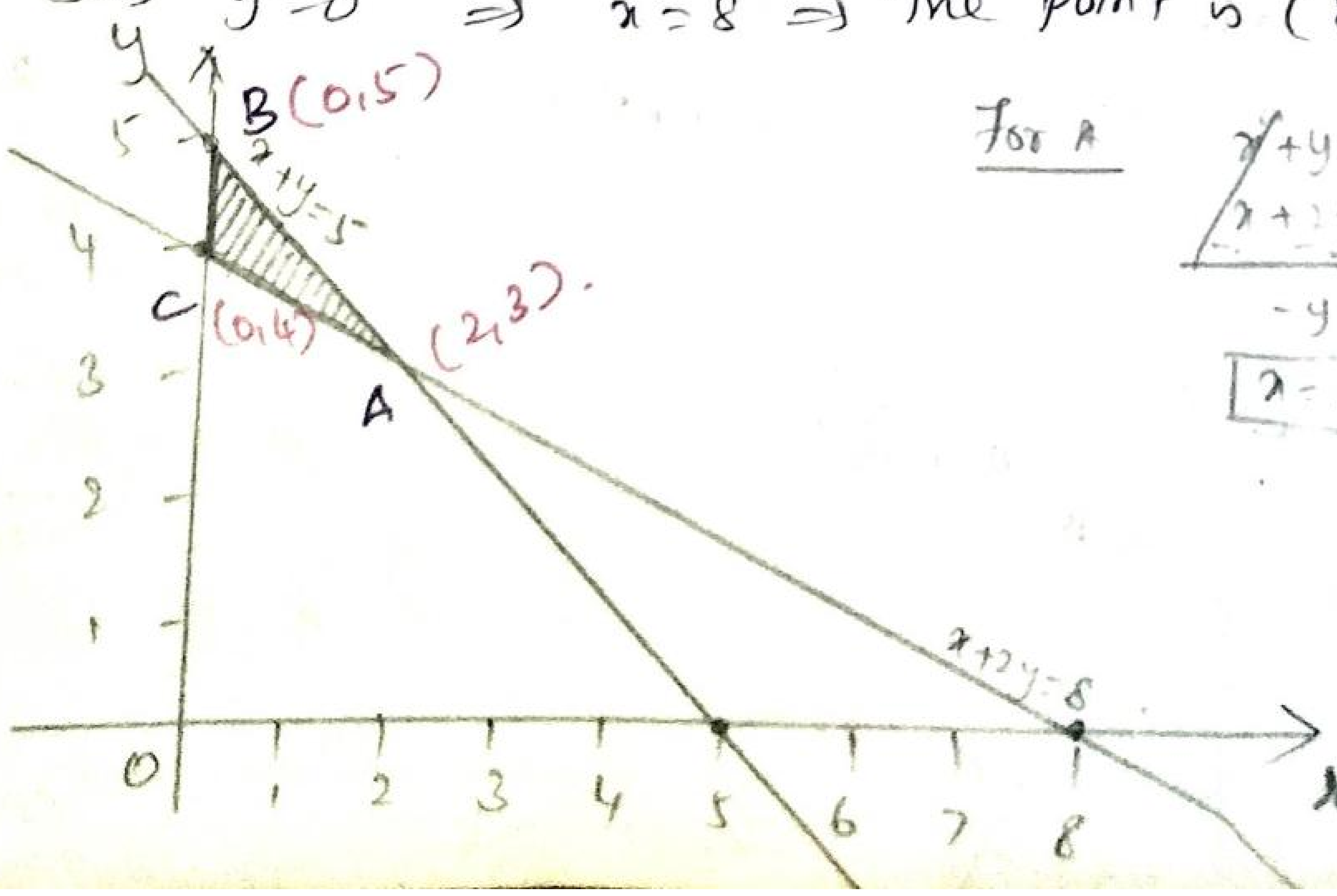
Sub $y = 0 \Rightarrow x = 5 \Rightarrow$ The point is $(5,0)$

(ii) Take $x + 2y \geq 8$.

$x + 2y = 8$.

Sub $x = 0 \rightarrow y = 4 \Rightarrow$ The point is $(0,4)$

Sub $y = 0 \Rightarrow x = 8 \Rightarrow$ The point is $(8,0)$.



For A

$x + y = 5$

$x + 2y = 8$

$-y = -3 \Rightarrow \boxed{y = 3}$

$\boxed{x = 2}$

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The feasible region is ABC

Extreme points	(x_1, x_2)	$z = 2x - y$
A	$(2,3)$	1
B	$(0,5)$	$-5 \leftarrow$ min.
C	$(0,4)$	-4

The optimum solution is

$$x_1 = 0, \quad x_2 = 5, \quad \min z = -5$$

7. $\max z = x_1 + x_2$

Subject to $\quad x_1 + x_2 \le 1$

$$-3x_1 + x_2 \ge 3$$

$$x_1, x_2 \ge 0.$$

Soln:

(i) Take $\quad x_1 + x_2 \le 1$

$$x_1 + x_2 = 1$$

Sub $x_1 = 0 \Rightarrow x_2 = 1 \Rightarrow$ The point is $(0,1)$

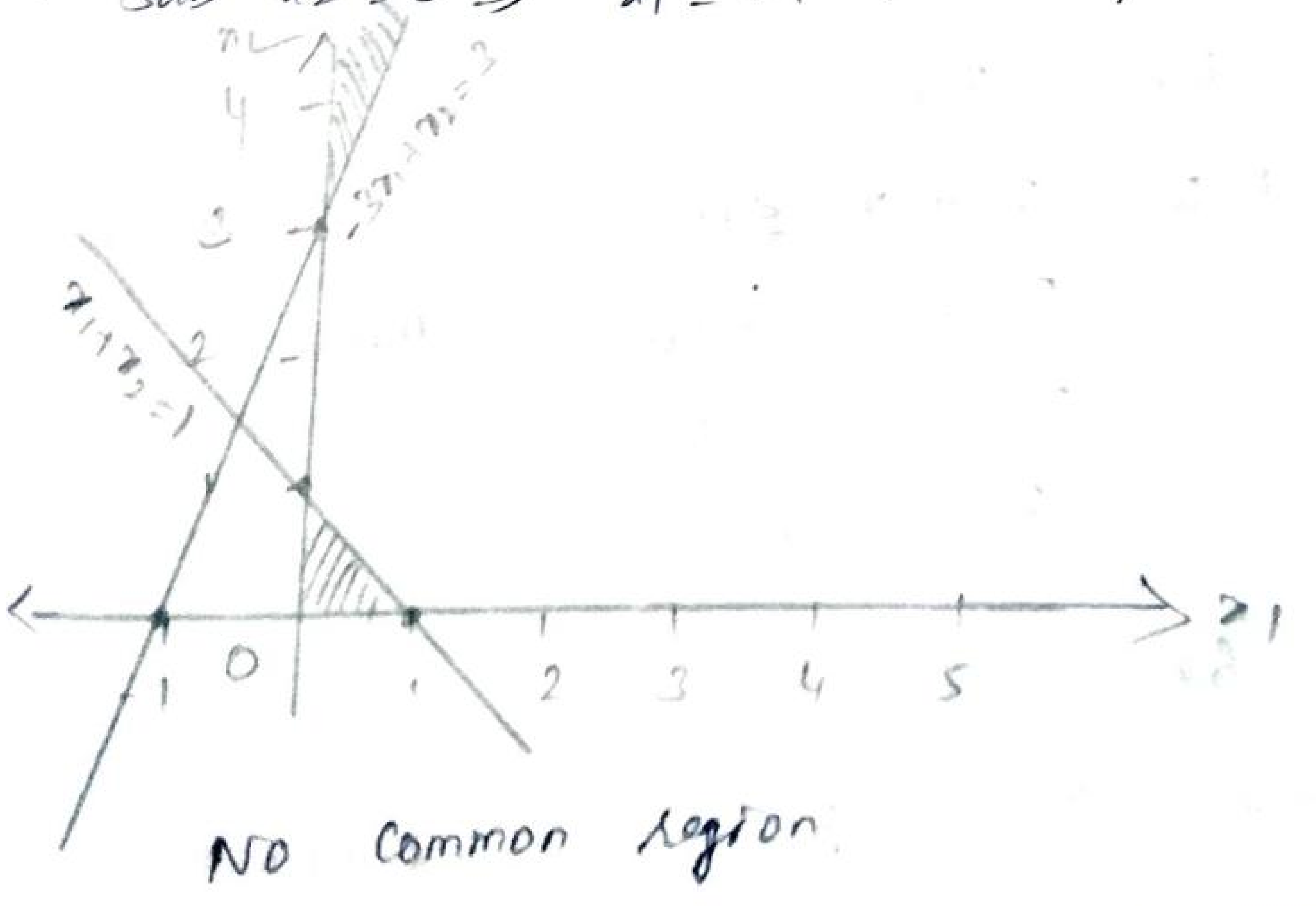
Sub $x_2 = 0 \Rightarrow x_1 = 1 \Rightarrow$ The point is $(1,0)$

(ii) Take $\quad -3x_1 + x_2 \ge 3$

$$-3x_1 + x_2 = 3.$$

Sub $x_1 = 0 \Rightarrow x_2 = 3 \Rightarrow$ The point is $(0,3)$

Sub $x_2 = 0 \Rightarrow x_1 = -1 \Rightarrow$ The point is $(-1,0)$



No common region.

$\therefore$ The given problem has no feasible solution.

Homework :

1) $\min z = -x_1 + 2x_2$

Subject to $-x_1 + 3x_2 \le 10$

$x_1 + x_2 \le 6$. Ans : $x_1 = 2$, $x_2 = 0$

$x_1 - x_2 \le 2$ $\min z = -2$.

$x_1, x_2 \ge 0$.

2. $\max z = 2x_1 + x_2$

Subject to $x_1 + 2x_2 \le 10$

$x_1 + x_2 \le 6$ Ans : $x_1 = 4$, $x_2 = 2$

$x_1 - x_2 \le 2$ $\max z = 10$.

$x_1 - 2x_2 \ge 10$

$x_1, x_2 \ge 0$.

3. Find the $\min z = 5x_1 - 2x_2$

Subject to $2x_1 + 3x_2 \ge 1$ $x_1 = 0$, $x_2 = 1/3$

$x_1, x_2 \ge 0$. $\min z = -2/3$.

4. $\min z = 2x_1 + x_2$

Subject to $5x_1 + 10x_2 \le 50$ $x_1 = 0$, $x_2 = 1$

$x_1 + x_2 \ge 1$ $\min z = 1$.

$x_1 \le 4$.

$x_1, x_2 \ge 0$.

5. $\min z = 4x_1 - 2x_2$.

Subject to $x_1 + x_2 \le 14$ $x_1 = 8$, $x_2 = 6$

$3x_1 + 2x_2 \ge 36$ $\min z = 20$.

$2x_1 + x_2 \le 24$

$x_1, x_2 \ge 0$.

6. $\max z = 6x_1 + x_2$

Subject to $2x_1 + x_2 \ge 3$ unbounded.

$x_1 - x_2 \ge 0$.

$x_1, x_2 \ge 0$

Simplex Method.

1. $\text{Max } z = 5x_1 + 3x_2$

Subject to
$$x_1 + x_2 \le 2$$
$$5x_1 + 2x_2 \le 10$$
$$3x_1 + 8x_2 \le 12$$
$$x_1, x_2 \ge 0$$

Soln:

By introducing the slack variables $x_3, x_4, x_5 \ge 0$ we get,

$$\max z = 5x_1 + 3x_2 + 0x_3 + 0x_4 + 0x_5$$

Subject to
$$x_1 + x_2 + x_3 = 2$$
$$5x_1 + 2x_2 + x_4 = 10.$$
$$3x_1 + 8x_2 + x_5 = 12.$$
$$x_1, x_2, x_3, x_4, x_5 \ge 0.$$

For finding the IBFS, sub $x_1 = x_2 = 0$,

we get $x_3 = 2$, $x_4 = 10$, $x_5 = 12$.

Initial Iteration:

			5	3	0	0	0	
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	$\theta = \min\left(\frac{x_B}{y_1}\right)$
0	y_3	2	1	1	1	0	0	$\frac{2}{1} = 2 \rightarrow$
0	y_4	10	5	2	0	1	0	$\frac{10}{5} = 2$
0	y_5	12	3	8	0	0	1	$\frac{12}{3} = 4$
$Z = C_B x_B = 0$,	$\Delta_j =$		$-5 \uparrow$	-3	0	0	0	

$\Delta_j = C_B y_B - C_j$

y_1 is the entering vector, y_3 is the removing vector

Iteration: 1

			5	3	0	0	0
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5
5	y_1	2	1	1	1	0	0
0	y_4	0	0	-3	-5	1	0
0	y_5	6	0	5	-3	0	1
$Z = 10$,	$\Delta_j =$		0	2	5	0	0

NSR = OSR − 5(NFR)

10 5 2 0 1 0 (−).
10 5 5 5 0 0

NTR = OTR − 3(NFR)

12 3 8 0 0 1
6 3 3 3 0 0
6 0 5 −3 0 1

All $\Delta_{ij} \geq 0$.

$\therefore$ The optimum solution is $x_1 = 2,\ x_2 = 0,\ \max z = 10$

2) $\max z = 5x_1 + 4x_2$

Subject to $4x_1 + 5x_2 \leq 10$.

$3x_1 + 2x_2 \leq 9$

$8x_1 + 3x_2 \leq 12$.

$x_1, x_2 \geq 0$.

Solution:

By introducing the slack variables $x_3, x_4, x_5 \geq 0$, we get

$$\max z = 5x_1 + 4x_2 + 0x_3 + 0x_4 + 0x_5.$$

Subject to

$$4x_1 + 5x_2 + x_3 = 10$$

$$3x_1 + 2x_2 + x_4 = 9$$

$$8x_1 + 3x_2 + x_5 = 12$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0.$$

For, finding IBFS, sub $x_1 = x_2 = 0$, we get $x_3 = 10$, $x_4 = 9$, $x_5 = 12$.

Initial Iteration:

			5	4	0	0	0	
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	θ
0	y_3	10	4	5	1	0	0	$10/4 = 2.5$
0	y_4	9	3	2	0	1	0	$9/3 = 3$
0	y_5	12	8	3	0	0	1	$12/8 = 1.5$ $\rightarrow$
$z = 0$,	$\Delta_{ij} =$		$-5 \uparrow$	-4	0	0	0	

y_1 is the entering vector, y_5 is the removing vector.

Iteration: 1

			5	4	0	0	0	
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	θ
0	y_3	4	0	$7/2$	1	0	$-1/2$	$8/7 = 1.1$ $\rightarrow$
0	y_4	$9/2$	0	$7/8$	0	1	$-3/8$	$\frac{36}{7} = 5.1$
5	y_1	$3/2$	1	$3/8$	0	0	$1/8$	4
$z = 15/2$	$\Delta_{ij} =$		0	$-17/8 \uparrow$	0	0	$5/8$	

NFR = OFR − 4 (NTR)

10	4	5	1	0	0
6	4	3/2	0	0	1/2
4	0	7/2	1	0	−1/2

(−)

NSR = OSR − 3 (NTR)

9	3	2	0	1	0
9/2	3	9/8	0	0	3/8
9/2	0	7/8	0	1	−3/8

(−)

Iteration: 2 y_2 is the entering vector, y_3 is the removing vector.

			5	4	0	0	0
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5
4	y_2	8/7	0	1	2/7	0	−1/7
0	y_4	7/2	0	0	−1/4	1	1/4
5	y_1	15/14	1	0	−3/28	0	5/28
$Z = 139/14$		$\Delta_{ij} =$	0	0	17/28	0	9/28

NSR = OSR − $\frac{7}{8}$ (NFR)

$\frac{9}{2}$	0	$\frac{7}{8}$	0	1	$-\frac{3}{8}$
1	0	$\frac{7}{8}$	$\frac{1}{4}$	0	−1/8
7/2	0	0	−1/4	1	1/4

(−)

NTR = OTR − $\frac{3}{8}$ (NFR)

$\frac{3}{2}$	1	$\frac{3}{8}$	0	0	$\frac{1}{8}$
3/7	0	3/8	$\frac{3}{28}$	0	$-\frac{3}{56}$
15/14	1	0	$-\frac{3}{28}$	0	$\frac{10}{56}$ 5/28

(−)

All $\Delta_{ij} \geq 0$.

∴ The optimum solution is $x_1 = \frac{15}{14}$, $x_2 = 8/7$.

$\max z = \frac{139}{14}$

3. Use simplex method to

$$\max Z = 10x_1 + x_2 + 2x_3.$$

subject to
$$x_1 + x_2 - 2x_3 \leq 10,$$
$$4x_1 + x_2 + x_3 \leq 20.$$
$$x_1, x_2, x_3 \geq 0.$$

Solution:

By Introducing slack variables $x_4, x_5 \geq 0$, we get

$$\max z = 10x_1 + x_2 + 2x_3 + 0x_4 + 0x_5$$

Subject to
$$x_1 + x_2 - 2x_3 + x_4 = 10$$
$$4x_1 + x_2 + x_3 + x_5 = 20$$
$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

For finding IBFS, sub $x_1 = x_2 = x_3 = 0$, we get

$x_4 = 10$, $x_5 = 20$.

Initial Iteration:

			10	1	2	0	0	
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	θ
0	y_4	10	1	1	-2	1	0	$10/1 = 10$
0	y_5	20	4	1	1	0	1	$20/4 = 5 \longrightarrow$
$Z = 0$,	$\Delta_j =$		-10 $\uparrow$	-1	-2	0	0	

y_1 is the entering vector, y_5 is the removing vector.

Iteration: 1

			10	1	2	0	0
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5
0	y_4	5	0	3/4	-9/4	1	-1/4
10	y_1	5	1	1/4	1/4	0	1/4
$Z = 50$,	$\Delta_j =$		0	3/2	1/2	0	5/2

NFR = OFR − 1(NSR)

10	1	1	-2	1	0
5	1	1/4	1/4	0	1/4
5	0	3/4	-9/4	1	-1/4

(−).

All $\Delta_j \geq 0$, The optimum solution is

$x_1 = 5$, $x_2 = 0$,

$\max Z = 50$.

4. $\max Z = x_1 - x_2 + 3x_3$.

Subject to
$$x_1 + x_2 + x_3 \leq 10,$$
$$2x_1 - x_3 \leq 2$$
$$2x_1 - 2x_2 + 3x_3 \leq 0$$
$$x_1, x_2, x_3 \geq 0.$$

Solution:

By introducing slack variables $x_4, x_5, x_6 \geq 0$

we get,

$\max z = x_1 - x_2 + 3x_3 + 0x_4 + 0x_5 + 0x_6$.

Subject to $\quad x_1 + x_2 + x_3 + x_4 = 10$.

$2x_1 - x_3 + x_5 = 2$

$2x_1 - 2x_2 + 3x_3 + x_6 = 0$.

For finding IBFS, sub $x_1 = x_2 = x_3 = 0$,

we get $x_4 = 10,\ x_5 = 2,\ x_6 = 0$.

Initial Iteration:

			1	-1	3	0	0	0	
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6	$\theta =$
0	y_4	10	1	1	1	1	0	0	$\frac{10}{1} = 10$
0	y_5	2	2	0	-1	0	1	0	–
0	y_6	0	2	-2	3	0	0	1	0 →
$Z =$ 0		$\Delta_j =$	-1	1	-3 ↑	0	0	0	

y_3 is the entering vector, y_6 is the removing vector.

Iteration: 1.

			1	-1	3	0	0	0
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6
0	y_4	10	1/3	5/3	0	1	0	-1/3
0	y_5	2	8/3	-2/3	0	0	1	1/3
3	y_3	0	2/3	-2/3	1	0	0	1/3
$Z =$ 0		$\Delta_j =$	1	-1 ↑	0	0	0	1

NFR = OFR − 1(NTR)

10	1	1	1	1	0	0
0	2/3	-2/3	1	0	0	1/3 (−)
10	1/3	5/3	0	1	0	-1/3

NSR = OSR + 1(NTR).

2	2	0	-1	0	1	0
0	2/3	-2/3	1	0	0	1/3
2	8/3	-2/3	0	0	1	1/3

y_2 is the entering vector,

y_4 is the removing vector.

Iteration: 2

			1	-1	3	0	0	0
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6
-1	y_2	6	1/5	1	0	3/5	0	-1/5
0	y_5	6	14/5	0	0	2/5	1	1/5
3	y_3	4	4/5	0	1	2/5	0	1/5
$Z = 6$,	$\delta_{ij} =$		$+\frac{16}{5}$	0	0	$\frac{3}{5}$	0	4/5

NSR = OSR + $\frac{2}{3}$ (NFR)

$$\begin{array}{ccccccc} 2 & \frac{8}{3} & -\frac{2}{3} & 0 & 0 & 1 & \frac{1}{3} \\ 4 & \frac{2}{15} & \frac{2}{3} & 0 & \frac{2}{5} & 0 & -\frac{2}{15} \\ \hline 6 & \frac{42}{15} & 0 & 0 & \frac{2}{5} & 1 & \frac{3}{15} \\ & \frac{14}{5} & & & & & \frac{1}{5} \end{array}$$

NTR = OTR + $\frac{2}{3}$ (NFR)

$$\begin{array}{ccccccc} 0 & \frac{2}{3} & -\frac{2}{3} & 1 & 0 & 0 & \frac{1}{3} \\ 4 & \frac{2}{15} & \frac{2}{3} & 0 & \frac{2}{5} & 0 & -\frac{2}{15} \\ \hline 4 & \frac{12}{15} & 0 & 1 & \frac{2}{5} & 0 & \frac{3}{15} \\ & \frac{4}{5} & & & & & \frac{1}{5} \end{array}$$

All $\delta_{ij} \geq 0$,

The optimum solution is $x_1 = 0, x_2 = 6, x_3 = 4$.

$\max z = 6$.

5. $\min z = x_2 - 3x_3 + 2x_5$

subject to $3x_2 - x_3 + 2x_5 \leq 7$.

$-2x_2 + 4x_3 \leq 12$

$-4x_2 + 3x_3 + 8x_5 \leq 10$

$x_2, x_3, x_5 \geq 0$.

Solution:

By introducing slack variables $x_6, x_7, x_8 \geq 0$, we get,

$\max z = -x_2 + 3x_3 - 2x_5 + 0x_6 + 0x_7 + 0x_8$.

Subject to $3x_2 - x_3 + 2x_5 + x_6 = 7$.

$-2x_2 + 4x_3 + x_7 = 12$

$-4x_2 + 3x_3 + 8x_5 + x_8 = 10$

$x_2, x_3, x_5, x_6, x_7, x_8 \geq 0$.

For finding IBFS, sub $x_2 = x_3 = x_5 = 0$,

we get, $x_6 = 7$, $x_7 = 12$, $x_8 = 10$.

Initial Iteration:

C_B	y_B	x_B	y_1 (-1)	y_2 (3)	y_3 (-2)	y_4 (0)	y_5 (0)	y_6 (0)	θ
0	y_4	7	3	-1	2	1	0	0	-
0	y_5	12	-2	4	0	0	1	0	3 →
0	y_6	10	-4	3	8	0	0	1	$\frac{10}{3} = 3.3$
$Z = 0$,	$\Delta_{ij} =$		1	-3 ↑	2	0	0	0	

y_2 is the entering vector, y_5 is the removing vector

Iteration: 1:

C_B	y_B	x_B	y_1 (-1)	y_2 (3)	y_3 (-2)	y_4 (0)	y_5 (0)	y_6 (0)	
0	y_4	10	5/2	0	2	1	1/4	0	→
3	y_2	3	-1/2	1	0	0	1/4	0	
0	y_6	1	-5/2	0	8	0	-3/4	1	
$Z = 9$	$\Delta_{ij} =$		-1/2 ↑	0	2	0	3/4	0	

NFR = OFR + 1 (NSR)

$$\begin{array}{ccccccc} 7 & 3 & -1 & 2 & 1 & 0 & 0 \\ 3 & -\frac{1}{2} & 1 & 0 & 0 & \frac{1}{4} & 0 \\ \hline 10 & \frac{5}{2} & 0 & 2 & 1 & \frac{1}{4} & 0 \end{array} \quad (+)$$

NFR = OFR − 3 (NSR).

$$\begin{array}{ccccccc} 10 & -4 & 3 & 8 & 0 & 0 & 1 \\ 9 & -\frac{3}{2} & 3 & 0 & 0 & \frac{3}{4} & 0 \\ \hline 1 & -\frac{5}{2} & 0 & 8 & 0 & -\frac{3}{4} & 1 \end{array} \quad (-)$$

y_1 is the entering vector, y_4 is the removing vector

Iteration: 2

C_B	y_B	x_B	y_1 (-1)	y_2 (3)	y_3 (-2)	y_4 (0)	y_5 (0)	y_6 (0)
-1	y_1	4	1	0	4/5	2/5	1/10	0
3	y_2	5	0	1	2/5	1/5	3/10	0
0	y_6	11	0	0	10	1	-1/2	1
$Z = 11$,	$\Delta_{ij} =$		0	0	12/5	1/5	8/10	0

$NSR = OSR + \frac{1}{2}(NFR)$

3	$-\frac{1}{2}$	1	0	0	1/4	0
2	1/2	0	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{1}{20}$	0
5	0	1	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{6}{20}$ $\frac{3}{10}$	0

(+)

$NTR = OTR + \frac{5}{2}(NFR)$

1	$-\frac{5}{2}$	0	8	0	$-\frac{3}{4}$	1
10	$\frac{5}{2}$	0	2	1	$\frac{1}{4}$	0
11	0	0	10	1	$-\frac{1}{2}$	1

(+)

All $\Delta_{ij} \geq 0$, The optimum solution is.

$x_2 = 4,\ x_3 = 5,\ x_5 = 0$

$\max z = 11,\quad \min z = -11.$

6. $\max z = 107x_1 + x_2 + 2x_3$. $\quad \frac{14}{3}x_1 + \frac{1}{3}x_2 - 2x_3 + x_4 = \frac{7}{3}$

Subject to $\quad 14x_1 + x_2 - 6x_3 + 3x_4 = 7 \Rightarrow$

$16x_1 + x_2 - 6x_3 \leq 5$

$3x_1 - x_2 - x_3 \leq 0$.

$x_1, x_2, x_3, x_4 \geq 0$.

Solution:

By introducing slack variables $x_5, x_6 \geq 0$, we get

$\max z = 107x_1 + x_2 + 2x_3 + 0x_4 + 0x_5 + 0x_6$

Subject to $\quad \frac{14}{3}x_1 + \frac{1}{3}x_2 - 2x_3 + x_4 = \frac{7}{3}$.

$16x_1 + x_2 - 6x_3 + x_5 = 5$

$3x_1 - x_2 - x_3 + x_6 = 0$.

$x_i \geq 0,\quad i = 1,2,3,4,5,6$

For finding IBFC, sub $x_1 = x_2 = x_3 = 0$,

we get $x_4 = \frac{7}{3},\ x_5 = 5,\ x_6 = 0$.

Initial Iteration:

			107	1	2	0	0	0	
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6	θ
0	y_4	7/3	14/3	1/3	−2	1	0	0	½ 0.5
0	y_5	5	16	1	−6	0	1	0	5/16 0.3
0	y_6	0	3	−1	−1	0	0	1	0 →
$z = 0$,		$\Delta_{ij} =$	−107 ↑	−1	−2	0	0	0	

y_1 is the entering vector, y_6 is the removing vector.

Iteration: 1

			107	1	2	0	0	0
C_B	y_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6
0	y_4	$7/3$	0	$17/9$	$-4/9$	1	0	$-14/9$
0	y_5	5	0	$19/3$	$-2/3$	0	1	$-16/3$
107	y_1	0	1	$-1/3$	$-1/3$	0	0	$1/3$
$Z = 0$,	$\Delta_j =$		0	$\frac{-110}{3}$	$\frac{-113}{3}$ ↑	0	0	$\frac{107}{3}$

$NFR = OFR - \frac{14}{3}(NTR)$

$\frac{7}{3}$	$\frac{14}{3}$	$\frac{1}{3}$	-2	1	0	0
(−) 0	$\frac{14}{3}$	$-\frac{14}{9}$	$-\frac{14}{9}$	0	0	$\frac{14}{9}$
$\frac{7}{3}$	0	$\frac{17}{9}$	$-\frac{4}{9}$	1	0	$-\frac{14}{9}$

$NSR = OSR - 16(NTR)$,

5	16	1	-6	0	1	0 (−)
0	16	$-\frac{16}{3}$	$-\frac{16}{3}$	0	0	$\frac{16}{3}$
5	0	$\frac{19}{3}$	$-\frac{2}{3}$	0	1	$-\frac{16}{3}$

y_3 is the entering vector. But all y_{i3} are negative

∴ The given problem has unbounded solution.

Homework:

1) $\max z = 3x_1 + 5x_2$

subject to $3x_1 + 2x_2 \leq 18$

$0 \leq x_1 \leq 4$

$0 \leq x_2 \leq 6.$

Ans: $x_1 = 2$
$x_2 = 6$
$\max z = 36$

2. $\max z = 2x_1 + 4x_2 + x_3 + x_4$

subject to $x_1 + 3x_2 + x_4 \leq 4$

$2x_1 + x_2 \leq 3$

$x_2 + 4x_3 + x_4 \leq 3$

$x_1, x_2, x_3, x_4 \geq 0.$

$x_1 = 1$
$x_2 = 1$
$x_3 = 1/2$
$x_4 = 0$
$\max z = 13/2.$

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