Equivalent Formulas

$P \vee P \Leftrightarrow P$ $\quad$ $P \wedge P \Leftrightarrow P$ [Idempotent laws]

$(P \vee Q) \vee R \Leftrightarrow P \vee (Q \vee R)$
$(P \wedge Q) \wedge R \Leftrightarrow P \wedge (Q \wedge R)$ } Associative Laws

$P \vee Q \Leftrightarrow Q \vee P$
$P \wedge Q \Leftrightarrow Q \wedge P$ } Commulative Laws

$P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$
$P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$ } Distributive Laws.

$P \vee F \Leftrightarrow P$ $\quad$ $P \wedge T \Leftrightarrow P$

$P \vee T \Leftrightarrow T$ $\quad$ $P \wedge F \Leftrightarrow F$

$P \vee \neg P \Leftrightarrow T$ & $P \wedge \neg P \Leftrightarrow F$

$P \vee (P \wedge Q) \Leftrightarrow P$
$P \wedge (P \vee Q) \Leftrightarrow P$ } Absorption Laws

$\neg (P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$
$\neg (P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$ } De Morgan's Law.

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Implications

$P \wedge Q \Rightarrow P$

$P \wedge Q \Rightarrow Q$

$P \Rightarrow P \vee Q$

$\neg P \Rightarrow P \to Q$

$Q \Rightarrow P \to Q$

$\neg(P \to Q) \Rightarrow P$

$\neg(P \to Q) \Rightarrow \neg Q$

$P \wedge (P \to Q) \Rightarrow Q$

$\neg Q \wedge (P \to Q) \Rightarrow \neg P$

$\neg P \wedge (P \vee Q) \Rightarrow Q$

$(P \to Q) \wedge (Q \to R) \Rightarrow P \to R$

$(P \vee Q) \wedge (P \to R) \wedge (Q \to R) \Rightarrow R$.

Unit II :- Normal Forms

Decision Problem :-

The Problem of determining, in a finite number of steps, whether a given statement formula is a tautology or a contradiction or at least satisfiable is known as a decision problem.

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Disjunctive Normal Forms

Conjunction → Product
Disjunction — Sum.

* A Product of the Variables and their negations in a formula is called an elementary product.

* A Sum of the variables and their negations in a formula is called an elementary sum.

Ex:- Let P and Q be any two atomic variables.

Then * P, $\neg P \wedge Q$, $\neg Q \wedge P$, $P \wedge \neg P$ are some elementary product.

* P, $\neg P \vee Q$, $\neg Q \vee P \vee \neg P$, $P \vee \neg P$ are some elementary sum.

* Any part of an elementary sum or elementary product which is itself an elementary sum or product is called factor of the original elementary sum or product.

Ex: $\neg Q$, $P \wedge \neg P$, $\neg Q \wedge P$ are some factors of $(\neg Q \wedge P \wedge \neg P)$

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* A necessary and sufficient condition for an elementary Product to be identically false is that it contain at least one pair of factors in which one is the negation of the other.

*. A necessary & sufficient Condition for an elementary Sum to be identically true is that it contain atleast one pair of factors in which one is the negation of the other.

Disjunctive Normal form

A formula which is equivalent to given formula and which consists of a sum of elementary products is called a DNF of the given formula.

$(\ \wedge\ ) \vee (\ \wedge\ )$

Procedure to obtain DNF

(i) Apply equivalent formula for '$\rightarrow$', '$\rightleftarrows$'

(ii) ~~Remove~~ Using De' Morgan's Law

(iii) Finally distributive laws, we get DNF.

Ex : 1

Obtain DNF of (a) $P \wedge (P \to Q)$

(b) $\neg(P \vee Q) \rightleftarrows (P \wedge Q)$

Soln :-

(a) $P \wedge (P \to Q) \Leftrightarrow P \wedge (\neg P \vee Q)$ $[\because P \to Q \Leftrightarrow \neg P \vee Q]$

$\Leftrightarrow (P \wedge \neg P) \vee (P \wedge Q)$ $[\because P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)]$

The DNF of $P \wedge (P \to Q)$ is $(P \wedge \neg P) \vee (P \wedge Q)$.

(b) $\neg(P \vee Q) \rightleftarrows (P \wedge Q)$

$\Leftrightarrow (\neg(P \vee Q) \wedge (P \wedge Q)) \vee (\neg\neg(P \vee Q) \wedge \neg(P \wedge Q))$

$[\because P \rightleftarrows Q \Leftrightarrow (P \wedge Q) \vee (\neg P \wedge \neg Q)]$

$\Leftrightarrow ((\neg P \wedge \neg Q) \wedge (P \wedge Q)) \vee ((P \vee Q) \wedge (\neg P \vee \neg Q))$

$\Leftrightarrow (\neg P \wedge \neg Q \wedge P \wedge Q) \vee ((P \vee Q) \wedge \neg P) \vee ((P \vee Q) \wedge \neg Q)$

$[\because P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)]$

$\Leftrightarrow (\neg P \wedge \neg Q \wedge P \wedge Q) \vee (P \wedge \neg P) \vee (Q \wedge \neg P) \vee (P \wedge \neg Q) \vee (Q \wedge \neg Q)$

which is the required disjunctive normal forms.

Note :-

~~$P \vee (Q \wedge R)$~~

$P \vee (Q \wedge R)$ → This formula is already in the DNF.

consider $P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$

$\Leftrightarrow (P \wedge P) \vee (P \wedge R) \vee (Q \wedge P) \vee (Q \wedge R)$

which is also DNF of $P \vee (Q \wedge R)$.

So DNF form is not unique

① 1.3.2 Conjunctive Normal Form

Defn :-

A formula which is equivalent to a given formula and which consists of a product of elementary sums is called a conjunctive normal form of the given formula

* The conjunctive normal form of the formula is not unique.

$(\vee) \wedge (\vee) \wedge (\vee)$

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Ex 1: Obtain CNF of
(a) $P \wedge (P \rightarrow Q)$ (b) $\neg(P \vee Q) \rightleftarrows (P \wedge Q)$

Soln:-

(a) $P \wedge (P \rightarrow Q) \Leftrightarrow P \wedge (\neg P \vee Q)$ [$\because P \rightarrow Q \Leftrightarrow \neg P \vee Q$]

Which is the required CNF.

(b) $\neg(P \vee Q) \rightleftarrows (P \wedge Q)$

Using the formula

$(P \rightleftarrows Q) \Leftrightarrow (\neg P \vee Q) \wedge (\neg Q \vee P)$

$\neg(P \vee Q) \rightleftarrows (P \wedge Q) \Leftrightarrow (\neg\neg(P \vee Q) \vee (P \wedge Q)) \wedge$
$(\neg(P \wedge Q) \vee \neg(P \vee Q))$

$\Leftrightarrow ((P \vee Q) \vee (P \wedge Q)) \wedge$
$((\neg P \vee \neg Q) \vee (\neg P \wedge \neg Q))$

$\Leftrightarrow (P \vee Q \vee P) \wedge (P \vee Q \vee Q) \wedge$
$(\neg P \vee \neg Q \vee \neg P) \wedge (\neg P \vee \neg Q \vee \neg Q)$

$\therefore$ The CNF of $\neg(P \vee Q) \rightleftarrows (P \wedge Q)$ is

$(P \vee Q \vee P) \wedge (P \vee Q \vee Q) \wedge (\neg P \vee \neg Q \vee \neg P) \wedge$

$(\neg P \vee \neg Q \vee \neg Q)$.

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Ex 2:-
Show that the formula
$Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$ is a Tautology.

Soln:-
First we obtain CNF of the given formula.

$Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$
$\Leftrightarrow Q \vee ((P \vee \neg P) \wedge \neg Q)$

$\Leftrightarrow (Q \vee P \vee \neg P) \wedge (Q \vee \neg Q) \to ①$

$(P \vee \neg P) \Leftrightarrow T$ & $(Q \vee \neg Q) \Leftrightarrow T$

$\therefore Q \vee (P \wedge \neg Q) \vee (\neg P \wedge \neg Q)$
$\Leftrightarrow (Q \vee T) \wedge T$

$\Leftrightarrow T \wedge T$ $[\because P \vee T \Leftrightarrow T]$

$\Leftrightarrow T$

$\therefore$ The given formula is a tautology.

1.3.3. Principal Disjunctive Normal Forms

Minterms :-

Let P and Q be two Variables, there are 2^2 formulas given by

$$P \wedge Q, \quad P \wedge \neg Q, \quad \neg P \wedge Q, \quad \neg P \wedge \neg Q.$$

These formulas are called minterms or Boolean Conjunctions of P and Q.

Similarly, let P, Q and R be three Variables, then the minterms are 2^3 terms

i.e,

$$P \wedge Q \wedge R, \quad P \wedge Q \wedge \neg R, \quad P \wedge \neg Q \wedge R, \quad P \wedge \neg Q \wedge \neg R$$

$$\neg P \wedge Q \wedge R, \quad \neg P \wedge Q \wedge \neg R, \quad \neg P \wedge \neg Q \wedge R, \quad \neg P \wedge \neg Q \wedge \neg R.$$

PDNF :-

For a given formula, an equivalent formula consisting of disjunctions of minterms only is known as its Principal disjunctive normal form, or Sum-of-products Canonical form.

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Truth Table Method 1 :-

Ex 1: Using truth tables find PDNF of $P \to Q$, $P \vee Q$, & $\neg(P \wedge Q)$, $P \rightleftarrows Q$.

Soln:-

~~Proof~~

PDNF of $(P \to Q)$:-

P	Q	¬P	¬Q	P∧Q	P∧¬Q	¬P∧Q	¬P∧¬Q	P→Q
T	T	F	F	$\boxed{T}$	F	F	F	$\boxed{T}$
T	F	F	T	F	T	F	F	F
F	T	T	F	F	F	$\boxed{T}$	F	$\boxed{T}$
F	F	T	T	F	F	F	$\boxed{T}$	$\boxed{T}$

$$(P \to Q) \Leftrightarrow (P \wedge Q) \vee (\neg P \wedge Q) \vee (\neg P \wedge \neg Q)$$

which is the required PDNF.

PDNF of $(P \vee Q)$

P	Q	¬P	¬Q	P∧Q	P∧¬Q	¬P∧Q	¬P∧¬Q	P∨Q
T	T	F	F	$\boxed{T}$	F	F	F	$\boxed{T}$
T	F	F	T	F	$\boxed{T}$	F	F	$\boxed{T}$
F	T	T	F	F	F	$\boxed{T}$	F	$\boxed{T}$
F	F	T	T	F	F	F	T	F

$$(P \vee Q) \Rightarrow (P \wedge Q) \vee (P \wedge \neg Q) \vee (\neg P \wedge Q)$$

PDNF of $\neg(P \wedge Q)$:-

P	Q	$\neg P$	$\neg Q$	$P \wedge Q$	$P \wedge \neg Q$	$\neg P \wedge Q$	$\neg P \wedge \neg Q$	$\neg(P \wedge Q)$
T	T	F	F	T	F	F	F	F
T	F	F	T	F	$\boxed{T}$	F	F	$\boxed{T}$
F	T	T	F	F	F	$\boxed{T}$	F	$\boxed{T}$
F	F	T	T	F	F	F	$\boxed{T}$	$\boxed{T}$

$$\neg(P \wedge Q) \Leftrightarrow (P \wedge \neg Q) \vee (\neg P \wedge Q) \vee (\neg P \wedge \neg Q)$$

which is the required PDNF.

PDNF of $(P \rightleftarrows Q)$:-

P	Q	$\neg P$	$\neg Q$	$P \wedge Q$	$P \wedge \neg Q$	$\neg P \wedge Q$	$\neg P \wedge \neg Q$	$P \rightleftarrows Q$
T	T	F	F	$\boxed{T}$	F	F	F	$\boxed{T}$
T	F	F	T	F	T	F	F	F
F	T	T	F	F	F	T	F	F
F	F	T	T	F	F	F	$\boxed{T}$	$\boxed{T}$

$$P \rightleftarrows Q \Leftrightarrow (P \wedge Q) \vee (\neg P \wedge \neg Q)$$

which is the required PDNF.

Method 2:- $(\wedge) \vee (\wedge)$

Ex2:- obtain the PDNF of

(a) $\neg P \vee Q$,

(b) $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$.

Soln:-

(a) $\neg P \vee Q \Leftrightarrow$

$\Leftrightarrow (\neg P \wedge T) \vee (Q \wedge T)$

$[\because P \wedge T \Leftrightarrow P]$

$\Leftrightarrow [\neg P \wedge (Q \vee \neg Q)] \vee [Q \wedge (P \vee \neg P)]$

$[\because P \vee \neg P \Leftrightarrow T]$

$\Leftrightarrow (\neg P \wedge Q) \vee (\neg P \wedge \neg Q) \vee (Q \wedge P) \vee (Q \wedge \neg P)$

[$\because$ Using distributive Law]

$\Leftrightarrow (\neg P \wedge Q) \vee (\neg P \wedge \neg Q) \vee (P \wedge Q) \vee (\neg P \wedge Q)$

$\Leftrightarrow (\neg P \wedge Q) \vee (\neg P \wedge \neg Q) \vee (P \wedge Q)$

which is the required PDNF.

~~$P \wedge \neg Q \Leftrightarrow (P \wedge Q) \vee (Q$~~

P	Q	¬P	¬Q	P∧Q	P∧¬Q	¬P∧Q	¬P∧¬Q	¬P∨Q
T	T	F	F	$\boxed{T}$	F	F	F	$\boxed{T}$
T	F	F	T	F	T	F	F	F
F	T	T	F	F	F	$\boxed{T}$	F	$\boxed{T}$
F	F	T	T	F	F	F	$\boxed{T}$	$\boxed{T}$

$\therefore \neg P \vee Q \Leftrightarrow (P \wedge Q) \vee (\neg P \wedge Q) \vee (\neg P \wedge \neg Q)$.

(b) $(P\wedge Q) \vee (\neg P\wedge R) \vee (Q\wedge R)$

This problem contains 3 variables P, Q & R.

$(P\wedge Q) \vee (\neg P\wedge R) \vee (Q\wedge R)$

$$\Leftrightarrow [(P\wedge Q)\wedge T] \vee [(\neg P\wedge R)\wedge T] \vee [(Q\wedge R)\wedge T]$$

$$\Leftrightarrow [(P\wedge Q)\wedge(R\vee\neg R)] \vee [(\neg P\wedge R)\wedge(Q\vee\neg Q)] \vee [(Q\wedge R)\wedge(P\vee\neg P)]$$

$$\Leftrightarrow (P\wedge Q\wedge R) \vee (P\wedge Q\wedge\neg R) \vee (\neg P\wedge R\wedge Q) \vee (\neg P\wedge R\wedge\neg Q) \vee (Q\wedge R\wedge P) \vee (Q\wedge R\wedge\neg P)$$

$$\Leftrightarrow (P\wedge Q\wedge R) \vee (P\wedge Q\wedge\neg R) \vee (\neg P\wedge Q\wedge R) \vee (\neg P\wedge\neg Q\wedge R) \vee (P\wedge Q\wedge R) \vee (\neg P\wedge Q\wedge R)$$

$$\Leftrightarrow (P\wedge Q\wedge R) \vee (P\wedge Q\wedge\neg R) \vee (\neg P\wedge Q\wedge R) \vee (\neg P\wedge\neg Q\wedge R)$$

[$\because$ write the repeating term only once]

$\therefore (P\wedge Q) \vee (\neg P\wedge R) \vee (Q\wedge R)$

$$\Leftrightarrow (P\wedge Q\wedge R) \vee (P\wedge Q\wedge\neg R) \vee (\neg P\wedge Q\wedge R) \vee (\neg P\wedge\neg Q\wedge R)$$

which is the required PDNF.

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1.3.4. Principal Conjunctive Normal Forms.

Maxterms :-
Let P and Q be two Variables, there are 2^2 formulas given by
$P \vee Q$, $P \vee \neg Q$, $\neg P \vee Q$, $\neg P \vee \neg Q$.
These formulas are called maxterms or Boolean disjunctions of P and Q.

\# The maxterms are the duals of the minterms.

PCNF :-
For a given formula, an equivalent formula consisting of conjunctions of the maxterms only is known as its Principal Conjunctive Normal form. This normal form is also called the Product - of - Sums canonical forms.

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Truth Table Method :-

Find the PCNF of $P \to Q$, $\neg P \wedge Q$, $P \rightleftarrows Q$.

Soln:-

Truth table Method :-

P	Q	$\neg P$	$\neg Q$	$P \vee Q$	$P \vee \neg Q$	$\neg P \vee Q$	$\neg P \vee \neg Q$	$P \to Q$	$\neg P \wedge Q$	$P \rightleftarrows Q$
T	T	F	F	T	T	T	$\boxed{F}$	T	$\boxed{F}$	T
T	F	F	T	T	T	$\boxed{F}$	T	$\boxed{F}$	$\boxed{F}$	$\boxed{F}$
F	T	T	F	T	$\boxed{F}$	T	T	T	T	$\boxed{F}$
F	F	T	T	$\boxed{F}$	T	T	T	T	$\boxed{F}$	T

The PCNF of.

$P \to Q \Leftrightarrow \neg P \vee Q$.

$\neg P \wedge Q \Leftrightarrow (\neg P \vee \neg Q) \wedge (\neg P \vee Q) \wedge (P \vee Q)$

$P \rightleftarrows Q \Leftrightarrow (\neg P \vee Q) \wedge (P \vee \neg Q)$.

Method 2:-

$P \to Q \Leftrightarrow \neg P \vee Q$, which is the required PCNF.

$\neg P \wedge Q \Leftrightarrow (\neg P \vee F) \wedge (Q \vee F)$ [$\because P \vee F \Leftrightarrow P$]

$\Leftrightarrow [\neg P \vee (Q \wedge \neg Q)] \wedge [Q \vee (P \wedge \neg P)]$ [$\because P \wedge \neg P \Leftrightarrow F$]

$\Leftrightarrow (\neg P \vee Q) \wedge (\neg P \vee \neg Q) \wedge (Q \vee P) \wedge (Q \vee \neg P)$ [$\because$ Distributive Law]

$\Leftrightarrow \underline{(\neg P \vee Q)} \wedge (\neg P \vee \neg Q) \wedge (P \vee Q) \wedge \underline{(\neg P \vee Q)}$

$\Leftrightarrow (\neg P \vee Q) \wedge (\neg P \vee \neg Q) \wedge (P \vee Q)$.

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$(\vee) \wedge (\vee)$

Example1:-

Obtain the PCNF of the formula $S: (\neg P \to R) \wedge (Q \rightleftarrows P)$. Also find the PDNF using PCNF.

Soln:-

$(\wedge) \vee (\wedge)$

$S: (\neg P \to R) \wedge (Q \rightleftarrows P)$

$\Leftrightarrow (\neg\neg P \vee R) \wedge ((Q \to P) \wedge (P \to Q))$

$\Leftrightarrow (P \vee R) \wedge (\neg Q \vee P) \wedge (\neg P \vee Q)$

$\Leftrightarrow [(P \vee R) \wedge F] \wedge [(\neg Q \vee P) \wedge F] \wedge [(\neg P \vee Q) \wedge F]$

$\Leftrightarrow [(P \vee R) \vee (Q \wedge \neg Q)] \wedge [(\neg Q \vee P) \vee (R \wedge \neg R)] \wedge [(\neg P \vee Q) \vee (R \wedge \neg R)]$

$\Leftrightarrow (P \vee R \vee Q) \wedge (P \vee R \vee \neg Q) \wedge (\neg Q \vee P \vee R) \wedge (\neg Q \vee P \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R)$

$\Leftrightarrow (P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R)$

$S: (P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R)$
$\wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R)$.

which is the required PCNF of the formula $(\neg P \rightarrow R) \wedge (Q \rightleftarrows P)$

To find PCNF of $\neg S$:-

The maxterms of P, Q, R is,

$\cancel{(P \vee Q \vee R)}$, $(P \vee Q \vee \neg R)$, $\cancel{(P \vee \neg Q \vee R)}$, $\cancel{(\neg P \vee Q \vee \neg R)}$
$\cancel{(P \vee \neg Q \vee \neg R)}$, $\cancel{(\neg P \vee Q \vee R)}$,
$(\neg P \vee \neg Q \vee R)$ $(\neg P \vee \neg Q \vee \neg R)$.

The P. conjunctive normal form of $\neg S$ can be obtained by writing the conjunction of the remaining maxterms.

$\therefore$ The PCNF of $\neg S$ is

$\neg S : (P \vee Q \vee \neg R) \wedge (\neg P \vee \neg Q \vee R) \wedge (\neg P \vee \neg Q \vee \neg R)$.

Also,

$S = \neg\neg S : \neg[(P \vee Q \vee \neg R) \wedge (\neg P \vee \neg Q \vee R) \wedge (\neg P \vee \neg Q \vee \neg R)]$

$\Leftrightarrow \neg(P \vee Q \vee \neg R) \vee \neg(\neg P \vee \neg Q \vee R) \vee \neg(\neg P \vee \neg Q \vee \neg R)$

$S \Leftrightarrow (\neg P \wedge \neg Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (P \wedge Q \wedge R)$

which the PDNF of given formula S.

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$(\wedge) \vee (\wedge)$

2. Find the PDNF of
$S : (\neg P \to R) \wedge (Q \rightleftarrows P)$

Soln:-

$S : (\neg P \to R) \wedge (Q \rightleftarrows P)$

$\Leftrightarrow (\neg\neg P \vee R) \wedge [(P \wedge Q) \vee (\neg P \wedge \neg Q)]$

$\Leftrightarrow (P \vee R) \wedge [(P \wedge Q) \vee (\neg P \wedge \neg Q)]$

$\Leftrightarrow (P \wedge (P \wedge Q)) \vee (P \wedge \neg P \wedge \neg Q) \vee (R \wedge P \wedge Q)$
$\vee (R \wedge \neg P \wedge \neg Q)$

$\Leftrightarrow (P \wedge Q) \vee (F \wedge \neg Q) \vee (P \wedge Q \wedge R) \vee$
$(\neg P \wedge \neg Q \wedge R)$ $[\because P \wedge \neg P \Leftrightarrow F]$

$\Leftrightarrow [(P \wedge Q) \wedge T] \vee F \vee (P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge R)$
$[\because P \wedge T = P$ & $P \wedge F = F]$

$\Leftrightarrow [(P \wedge Q) \wedge (R \vee \neg R)] \vee (P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge R)$

$\Leftrightarrow (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (P \wedge Q \wedge R) \vee (\neg P \wedge \neg Q \wedge \neg R)$

$\Leftrightarrow (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \neg R) \vee (\neg P \wedge \neg Q \wedge \neg R)$

which is the required PDNF of the given formula S.

$(P \vee Q \vee R) \wedge (P \vee Q \vee \neg R) \wedge (\neg P \vee Q \vee \neg R)$
$\wedge$ ~~$(P \wedge \neg Q)$~~ $(P \vee \neg Q \vee R) \wedge (\neg P \vee Q \vee R)$

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1.3.5 :- Ordering & Uniqueness of Normal Forms.

Minterm :-

* Given any n statement variables, let us first arrange them in some fixed order.
* The 2^n minterms corresponding to the n variables can be designated by $m_0, m_1, m_2, \ldots, m_{2^n-1}$

* If in the i^{th} location from the left there appears 1 then the i^{th} variable appears in the conjunction.

* If 0 appears in the i^{th} location then the negation of the i^{th} variable appears in the conjunction forming the minterm.

* Let P, Q, R be three variables arranged in that order. The minterms are denoted by $m_0, m_1, m_2, \ldots m_7$.

$m_0 \rightarrow 000 \rightarrow \neg P \wedge \neg Q \wedge \neg R$

$m_1 \rightarrow 001 \rightarrow \neg P \wedge \neg Q \wedge R$

$m_2 \rightarrow 010 \rightarrow \neg P \wedge Q \wedge \neg R$

$m_3 \rightarrow 011 \rightarrow \neg P \wedge Q \wedge R$

$m_4 \rightarrow 100 \rightarrow P \wedge \neg Q \wedge \neg R$

$m_5 \rightarrow 101 \rightarrow P \wedge \neg Q \wedge R$

$m_6 \rightarrow 110 \rightarrow P \wedge Q \wedge \neg R$

$m_7 \rightarrow 111 \rightarrow P \wedge Q \wedge R$.

2 | 4
2 | 2 − 0
  1 − 0

2 | 5
2 | 2 − 1
  1 − 0

2 | 6
2 | 3 − 0
  1 − 1

2 | 7
2 | 3 − 1
  1 − 1

\# Let $P_1, P_2, \ldots P_6$ be six variables arranged in that order. The minterms are denoted by $m_0, m_1, \ldots m_{2^6-1}$ i.e, $m_0, m_1, \ldots m_{63}$.

Find m_{35} :-

$1 \rightarrow P$

$0 \rightarrow \neg P$

2 | 35
2 | 17 − 1
2 | 8 − 1
2 | 4 − 0
2 | 2 − 0
  1 − 0

1 0 0 0 1 1

$\therefore m_{35} = P_1 \wedge \neg P_2 \wedge \neg P_3 \wedge \neg P_4 \wedge P_5 \wedge P_6$

Find m_8

2 | 8
2 | 4 − 0
2 | 2 − 0
  1 − 0

001000
(adding zero on left)

$\therefore$ $m_8 = \neg P_1 \wedge \neg P_2 \wedge P_3 \wedge \neg P_4 \wedge \neg P_5 \wedge \neg P_6$.

To find m_{53} :-

$$
\begin{array}{r|l}
2 & 53 \\
2 & 26 - 1 \\
2 & 13 - 0 \\
2 & 6 - 1 \\
2 & 3 - 0 \\
 & 1 - 1
\end{array}
$$

$1\ 1\ 0\ 1\ 0\ 1$

$\therefore m_{53} = P_1 \wedge P_2 \wedge \neg P_3 \wedge P_4 \wedge \neg P_5 \wedge P$

~~Maxterms~~ :-

* Using this notation, the sum-of-product canonical form representing the disjunction of m_i, m_j and m_k

ie, The PDNF of $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$ is

$$\underbrace{(P \wedge Q \wedge R)}_{m_7} \vee \underbrace{(P \wedge Q \wedge \neg R)}_{m_6} \vee \underbrace{(\neg P \wedge Q \wedge R)}_{m_3} \vee \underbrace{(\neg P \wedge \neg Q \wedge R)}_{m_1}$$

i.e, $\Sigma\ 1, 3, 6, 7$.

Maxterms :-

The 2^n maxterms corresponding to the n variables can be designated by $M_0, M_1, M_2, \dots M_{2^n-1}$.

* If 1 appears in the i^{th} location, then the negation of the i^{th} variable appears in the ~~conjunction~~ disjunction.
* If 0 appears in the i^{th} location then the i^{th} variable appears in the disjunction.

* Let P, Q and R are three variables arranged in that order. The Maxterms are denoted by $M_0, M_1, \dots M_7$.

$M_0 \to 000 \to P \vee Q \vee R$
$M_1 \to 001 \to P \vee Q \vee \neg R$
$M_2 \to 010 \to P \vee \neg Q \vee R$
$M_3 \to 011 \to P \vee \neg Q \vee \neg R$
$M_4 \to 100 \to \neg P \vee Q \vee R$
$M_5 \to 101 \to \neg P \vee Q \vee \neg R$
$M_6 \to 110 \to \neg P \vee \neg Q \vee R$
$M_7 \to 111 \to \neg P \vee \neg Q \vee \neg R$.

Let P_1, P_2, P_3, P_4, P_5 be five Variables arrange in that order. The Maxterms are denoted by,
$M_0, M_1, M_2, \ldots M_{2^5-1}$
ie, $M_0, M_1, M_2, \ldots M_{31}$

Find M_{26}

$1 \to \neg P$
$0 \to P$

$$\begin{array}{r|l} 2 & 26 \\ 2 & 13-0 \\ 2 & 6-1 \\ 2 & 3-0 \\ & 1-1 \end{array}$$

1 1 0 1 0
$\neg P_1 \vee \neg P_2 \vee P_3 \vee \neg P_4 \vee P_5$.

M_{10} :-

$$\begin{array}{r|l} 2 & 10 \\ 2 & 5-0 \\ 2 & 2-1 \\ & 1-0 \end{array}$$

0 1 0 1 0 (adding zero on left)
~~$P_1 P_2$~~
$P_1 \vee \neg P_2 \vee P_3 \vee \neg P_4 \vee P_5$

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$(\vee) \wedge (\vee)$

Ex:- obtain PCNF of $(P\wedge Q) \vee (\neg P \wedge R)$

Soln:-

$(P\wedge Q) \vee (\neg P\wedge R)$

$\Leftrightarrow (P\vee \neg P) \wedge (P\vee R) \wedge (Q\vee \neg P) \wedge (Q\vee R)$

$\Leftrightarrow T \wedge (P\vee R) \wedge (\neg P\vee Q) \wedge (Q\vee R)$

$\Leftrightarrow [(P\vee R)\vee F] \wedge [(\neg P\vee Q)\vee F] \wedge [(Q\vee R)\vee F]$

$\Leftrightarrow [(P\vee R)\vee (Q\wedge \neg Q)] \wedge [(\neg P\vee Q)\vee (R\wedge \neg R)] \wedge [(Q\vee R)\vee (P\wedge \neg P)]$

$\Leftrightarrow (P\vee R\vee Q) \wedge (P\vee R\vee \neg Q) \wedge (\neg P\vee Q\vee R) \wedge (\neg P\vee Q\vee \neg R) \wedge (Q\vee R\vee P) \wedge (Q\vee R\vee \neg P)$

$\Leftrightarrow (P\vee Q\vee R) \wedge (P\vee \neg Q\vee R) \wedge (\neg P\vee Q\vee R)$ $(\neg P\vee Q\vee \neg R) \wedge (P\vee Q\vee R) \wedge (\neg P\vee Q\vee R)$

$\Leftrightarrow \underset{M_0}{(P\vee Q\vee R)} \wedge \underset{M_2}{(P\vee \neg Q\vee R)} \wedge \underset{M_4}{(\neg P\vee Q\vee R)} \wedge (\neg P\vee Q\vee \neg R) \rightarrow M_5$

$\Leftrightarrow \Pi\, 0, 2, 4, 5.$

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PDNF of $S: (P \wedge Q) \vee \neg P \wedge R)$.

Soln:-

$S: (P \wedge Q) \vee (\neg P \wedge R)$

The PCNF of S is

$$S: (P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R)$$

$$\neg S: (P \vee Q \vee \neg R) \wedge (P \vee \neg Q \vee \neg R) \wedge (\neg P \vee \neg Q \vee \neg R) \wedge (\neg P \vee \neg Q \vee R)$$

$$S = \neg\neg S : \neg\left[(P \vee Q \vee \neg R) \wedge (P \vee \neg Q \vee \neg R) \wedge (\neg P \vee \neg Q \vee \neg R) \wedge (\neg P \vee \neg Q \vee R)\right]$$

$$\Leftrightarrow \neg(P \vee Q \vee \neg R) \vee \neg(P \vee \neg Q \vee \neg R) \vee \neg(\neg P \vee \neg Q \vee \neg R) \vee \neg(\neg P \vee \neg Q \vee R)$$

$$\Leftrightarrow \underset{m_1}{(\neg P \wedge \neg Q \wedge R)} \vee \underset{m_3}{(\neg P \wedge Q \wedge R)} \vee \underset{m_7}{(P \wedge Q \wedge R)} \vee \underset{m_6}{(P \wedge Q \wedge \neg R)}$$

$$\Leftrightarrow \Sigma\, 1, 3, 6, 7.$$

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1.4.1. Validity Using Truth Tables.

Let A and B be two statement formulas. We say that "B logically follows from A" or "B is a valid conclusion of the Primise A" iff $(A \rightarrow B)$ is a tautology, ie, $A \Rightarrow B$.

Note :-

A set of Premises $\{H_1, H_2, \cdots H_m\}$ is a conclusion of C follows logically if

$$H_1 \wedge H_2 \wedge H_3 \wedge \cdots \wedge H_m \Rightarrow C.$$

Exam Determine whether the conclusion C follows logically from the Primises H_1 and H_2.

(a) $H_1 : P \rightarrow Q$ $\quad H_2 : P$ $\quad C : Q$

(b) $H_1 : P \rightarrow Q$ $\quad H_2 : \neg P$ $\quad C : Q$

(c) $H_1 : P \rightarrow Q$ $\quad H_2 : \neg(P \wedge Q)$ $\quad C : \neg P$

(d) $H_1 : \neg P$ $\quad H_2 : P \rightleftarrows Q$ $\quad C : \neg(P \wedge Q)$

(e) $H_1 : P \rightarrow Q$ $\quad H_2 : Q$ $\quad C : P.$

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Soln:-

(a) $H_1 : P \to Q$ $H_2 : P$ $C : Q$.

P	Q	$P \to Q$ (H_1)	P (H_2)	$(P \to Q) \wedge P$ ($H_1 \wedge H_2$)	Q (C)	$[(P \to Q) \wedge P] \to Q$ ($(H_1 \wedge H_2) \to P$)
T	T	T	T	T	T	T
T	F	F	T	F	F	T
F	T	T	F	F	T	T
F	F	T	F	F	F	T

$[(P \to Q) \wedge P] \to Q$ is a Tautology.

$\therefore$ The conclusion $C : Q$ logically follows from the Premises $H_1 : P \to Q$ and $H_2 : P$.

(b) $H_1 : P \to Q$ $H_2 : \neg P$ $C : Q$.

P	Q	$P \to Q$ (H_1)	$\neg P$ (H_2)	$(P \to Q) \wedge \neg P$ ($H_1 \wedge H_2$)	Q (C)	$[(P \to Q) \wedge \neg P] \to Q$ ($(H_1 \wedge H_2) \to C$)
T	T	T	F	F	T	T
T	F	F	F	F	F	T
F	T	T	T	T	T	T
F	F	T	T	T	F	F

$[(P \to Q) \wedge \neg P] \to Q$ is not a Tautology

$\therefore$ ~~The conclusion~~ Q is not ~~logically~~ a valid Conclusion of $(P \to Q)$ and $\neg P$.

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c) $H_1: P \to Q$ $H_2: \neg(P \land Q)$ $C: \neg P$

		H_1		H_2	$H_1 \land H_2$	C	$(H_1 \land H_2) \to C$
P	Q	$P \to Q$	$P \land Q$	$\neg(P \land Q)$	$(P \to Q) \land \neg(P \land Q)$	$\neg P$	$[(P \to Q) \land \neg(P \land Q)] \to \neg P$
T	T	T	T	F	F	F	T
T	F	F	F	T	F	F	T
F	T	T	F	T	T	T	T
F	F	T	F	T	T	T	T

$\therefore [(P \to Q) \land \neg(P \land Q)] \to \neg P$ is Tautology

$\therefore$ $\neg P$ is a valid conclusion of the premises $(P \to Q)$ and $\neg(P \land Q)$.

d) $H_1: \neg P$ $H_2: P \rightleftarrows Q$ $C: \neg(P \land Q)$

		H_1	H_2	$H_1 \land H_2$		C	$(H_1 \land H_2) \to C$
P	Q	$\neg P$	$P \rightleftarrows Q$	$\neg P \land (P \rightleftarrows Q)$	$P \land Q$	$\neg(P \land Q)$	$[\neg P \land (P \rightleftarrows Q)] \to \neg(P \land Q)$
T	T	F	T	F	T	F	T
T	F	F	F	F	F	T	T
F	T	T	F	F	F	T	T
F	F	T	T	T	F	T	T

$\therefore [\neg P \land (P \rightleftarrows Q)] \to \neg(P \land Q$ is a Tautology.

$\therefore$ The conclusion C is valid from the premises $\neg P$ and $P \rightleftarrows Q$.

(e) $H_1: P \to Q$ $H_2: Q$, $C: P$

P	Q	$P \to Q$	Q	$(P \to Q) \wedge Q$	P	$[(P \to Q) \wedge Q] \to P$
T	T	T	T	T	T	T
T	F	F	F	F	T	T
F	T	T	T	T	F	F
F	F	T	F	F	F	T

$\therefore [(P \to Q) \wedge Q] \to P$ is not a Tautology.

$\therefore$ The conclusion is not a Valid.

1.4.2. Rules of Inference.

Rule P : A premise may be introduced at any point in ~~that~~ the derivation.

Rule T : A formula S may be introduced in a derivation if S is logically implied by any one or more of the preceding formulas in the derivation.

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Implications

I_1 $P \wedge Q \Rightarrow P$ } Simplification
I_2 $P \wedge Q \Rightarrow Q$
I_3 $P \Rightarrow P \vee Q$ } addition
I_4 $Q \Rightarrow P \vee Q$
I_5 $\neg P \Rightarrow P \rightarrow Q$
I_6 $Q \Rightarrow P \rightarrow Q$
I_7 $\neg(P \rightarrow Q) \Rightarrow P$
I_8 $\neg(P \rightarrow Q) \Rightarrow \neg Q$
I_9 $P, Q \Rightarrow P \wedge Q$
I_{10} $\neg P, P \vee Q \Rightarrow Q$ ✓ (disjunctive syllogism)
I_{11} $P, P \rightarrow Q \Rightarrow Q$ ✓ (modus ponens)
I_{12} $\neg Q, P \rightarrow Q \Rightarrow \neg P$ (modus tollens)
I_{13} $P \rightarrow Q, Q \rightarrow R \Rightarrow P \rightarrow R$ ✓ (hypothetical Syllogism)
I_{14} $P \vee Q, P \rightarrow R, Q \rightarrow R \Rightarrow R$ (dilemma)

Equivalences

E_1 $\neg\neg P \Leftrightarrow P$ ✓ (double negation)
E_2 $P \wedge Q \Leftrightarrow Q \wedge P$ } ✓ Commutative laws
E_3 $P \vee Q \Leftrightarrow Q \vee P$

E_4 $(P \wedge Q) \wedge R \Leftrightarrow P \wedge (Q \wedge R)$ } ✓ associative laws
E_5 $(P \vee Q) \vee R \Leftrightarrow P \vee (Q \vee R)$

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E6 $P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$ } distributive laws
E7 $P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \wedge R)$

E8 $\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q$ } De Morgon's Laws
E9 $\neg(P \vee Q) \Leftrightarrow \cancel{\neg P \wedge Q}\ \neg P \wedge \neg Q$

E10 $P \vee P \Leftrightarrow P$
E11 $P \wedge P \Leftrightarrow P$
E12 $R \vee (P \wedge \neg P) \Leftrightarrow R$
E13 $R \wedge (P \vee \neg P) \Leftrightarrow R$
E14 $R \vee (P \vee \neg P) \Leftrightarrow T$
E15 $R \wedge (P \wedge \neg P) \Leftrightarrow F$

E16 $P \rightarrow Q \Leftrightarrow \neg P \vee Q$ ✓
E17 $\neg(P \rightarrow Q) \Leftrightarrow P \wedge \neg Q$

E18 $P \rightarrow Q \Leftrightarrow \neg Q \rightarrow \neg P$ ✓

E19 $P \rightarrow (Q \rightarrow R) \Leftrightarrow (P \wedge Q) \rightarrow R \Leftrightarrow \underline{Q \rightarrow (P \rightarrow R)}$

E20 $\neg(P \rightleftarrows Q) \Leftrightarrow (P \rightleftarrows \neg Q)$

E21 $(P \rightleftarrows Q) \Leftrightarrow (P \rightarrow Q) \wedge (Q \rightarrow P)$ ✓

E22 $(P \rightleftarrows Q) \Leftrightarrow (P \wedge Q) \vee (\neg P \wedge \neg Q)$. ✓

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Example 1:

Demonstrate that R is a Valid inference from the Premises $P \to Q$, $Q \to R$ and P.

Soln:-

{1}	1.	$P \to Q$	Rule P
{2}	2.	P	Rule P
{1,2}	3.	Q	Rule T $P, P \to Q \Rightarrow Q$
{4}	4.	$Q \to R$	Rule P
{1,2,5}	5.	R	Rule T $P, P \to Q \Rightarrow Q$

(OR)

{1}	1.	$P \to Q$	Rule P
{2}	2.	$Q \to R$	Rule P
{1,2}	3.	$P \to R$	Rule T $P \to Q, Q \to R \Rightarrow P \to R$
{4}	4.	P	Rule P
{1,2,4}	5.	R	Rule T $P, P \to Q \Rightarrow Q$

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Ex 2:- Show that $R \vee S$ follows logically from the premises $C \vee D$, $(C \vee D) \to \neg H$, $\neg H \to (A \wedge \neg B)$ and $(A \wedge \neg B) \to (R \vee S)$

Soln:-

{1}	1.	$(C \vee D) \to \neg H$	Rule P
{2}	2.	$\neg H \to (A \wedge \neg B)$	Rule P
{1,2}	3.	$(C \vee D) \to (A \wedge \neg B)$	Rule T $P \to Q, Q \to R \Rightarrow P \to R$
{4}	4.	$(A \wedge \neg B) \to (R \vee S)$	Rule P
{1,2,4}	5.	$(C \vee D) \to R \vee S$	Rule T $P \to Q, Q \to R \Rightarrow P \to R$
{6}	6.	$C \vee D$	Rule P
{1,2,4,6}	7.	$R \vee S$	Rule T $P, P \to Q \Rightarrow Q$.

(OR)

{1}	1.	$C \vee D$	Rule P
{2}	2.	$(C \vee D) \to \neg H$	Rule P
{1,2}	3.	$\neg H$	Rule T $P, P \to Q \Rightarrow Q$
{4}	4.	$\neg H \to (A \wedge \neg B)$	Rule P
{1,2,4}	5.	$(A \wedge \neg B)$	Rule T $P, P \to Q \Rightarrow Q$
{6}	6.	$(A \wedge \neg B) \to (R \vee S)$	Rule P
{1,2,4,6}	7.	$(R \vee S)$	Rule T $P, P \to Q \Rightarrow Q$.

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EX 3 Show that $(S \vee R)$ is tautologically implied by $(P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow S)$

Soln :—

{1}	1.	$P \vee Q$	Rule P
	~~2.~~	~~$\neg P \rightarrow Q$~~	~~Rule T~~
{1}	2.	$\neg P \rightarrow Q$	Rule T $P \rightarrow Q \Leftrightarrow (\neg P \vee Q)$
{3}	3.	$Q \rightarrow S$	Rule P
{1,3}	4.	$\neg P \rightarrow S$	Rule T $P \rightarrow Q, Q \rightarrow R \Rightarrow P \rightarrow R$
{1,3}	5.	$\neg S \rightarrow P$	Rule T $(P \rightarrow Q) \Leftrightarrow (\neg Q \rightarrow \neg P)$
{6}	6.	$P \rightarrow R$	Rule P
{1,3,6}	7.	$\neg S \rightarrow R$	Rule T $P \rightarrow Q, Q \rightarrow R \Rightarrow P \rightarrow R$
{1,3,6}	8.	$S \vee R$	Rule T $P \rightarrow Q \Leftrightarrow \neg Q \rightarrow \neg P$

Ex:4 Show that $R \wedge (P \vee Q)$ is a valid conclusion from the premises $P \vee Q$, $Q \rightarrow R$, $P \rightarrow M$ and $\neg M$.

Soln:-

{1}	1. $P \rightarrow M$	Rule P	
{2}	2. $\neg M$	Rule P	
{1,2}	3. $\neg P$	Rule T	$\neg Q, P \rightarrow Q \Rightarrow \neg P$
{4}	4. $P \vee Q$	Rule P	
{1,2,4}	5. Q	Rule T	$\neg P, P \vee Q \Rightarrow Q$
{6}	6. $Q \rightarrow R$	Rule P	
{1,2,4,6}	7. R	Rule T	$P, P \rightarrow Q \Rightarrow Q$
{1,2,4,6}	8. $R \wedge (P \vee Q)$	Rule T	$P, Q \Rightarrow P \wedge Q$

Ex 5: Show I_{12}: $\neg Q, P \rightarrow Q \Rightarrow \neg P$

Soln:-

{1}	1 $P \rightarrow Q$	Rule P	
{1}	2. $\neg Q \rightarrow \neg P$	Rule T	$(P \rightarrow Q) \Leftrightarrow \neg Q \rightarrow \neg P$
{3}	3. $\neg Q$	Rule P	
{1,3}	4. $\neg P$	Rule T	$P, P \rightarrow Q \Rightarrow Q$

Rule of Conditional Proof or Rule CP

Rule CP: If we can derive S from R and a set of premises, then we can derive $R \rightarrow S$ from the set of premises alone

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i.e, Rule CP is also called the deduction theorem and is generally used if the conclussion is of the form $R \to S$. In such cases, R is taken an as additional Premise and S is derived from the given premises and R.

Ex 6:- Show that $R \to S$ can be derived from the premises $P \to (Q \to S)$, $\neg R \vee P$, and Q.

Soln:- Instead of deriving $R \to S$, we shall include R as an additional Premise and show S first.

$\{1\}$	1	$\neg R \vee P$	Rule P
$\{2\}$	2.	R	Rule P (assumed premise)
$\{1,2\}$	3.	P	Rule T $R, \neg R \vee P \Rightarrow P$
$\{4\}$	4.	$P \to (Q \to S)$	Rule P
$\{1,2,4\}$	5.	$Q \to S$	Rule T $P, P \to Q \Rightarrow Q$
$\{6\}$	6.	Q	Rule P
$\{1,2,4,6\}$	7.	S	Rule T $P, P \to Q \Rightarrow Q$
$\{1,4,6\}$	8.	$R \to S$	Rule CP.

Ex 7: "If there was a ball game then travelling was difficult. If they arrived on time then travelling was not difficult. They arrived on time. Therefore, there was no ball game".
Show that those statements constitute a valid argument.

Soln:-

Let P : There was a ball game
$\neg P$: There was no ball game.
Q : Travelling was difficult
$\neg Q$: Travelling was not difficult.
R : They arrived on time.

Premises:-

If there was a ball game then travelling was difficult. : $P \to Q$

If they arrived on time then travelling was not difficult:
$R \to \neg Q$.

They arrived on time : R.

Conclusion :-

There was no ball game : $\neg P$.

{1}	1.	$R \to \neg Q$	Rule P	
{2}	2	R	Rule P	
{1,2}	3.	$\neg Q$	Rule T	$P, P \to Q \Rightarrow Q$
{4}	4.	$P \to Q$	Rule P	
{4}	5.	$\neg Q \to \neg P$	Rule T	$P \to Q \Leftrightarrow \neg Q \to \neg P$
{1,2,4}	6.	$\neg P$	Rule T	$P, P \to Q \Rightarrow Q$.

Example 8:- If A works hard then B or C will enjoy them selves. If B enjoys himself then A will not work hard. If D enjoys himself, then C will not. There fore if A works hard, D will not enjoy him self.

Soln :-

Let : A : A works hard

B : B will enjoy himself

C : C will enjoy him self.

D : D will enjoy him self.

Premises:-

If A works hard, then B or C will enjoy themselves : $A \to (B \vee C)$

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If B enjoys himself then A will not work hard : $B \to \neg A$

If D enjoys himself then C will not
$D \to \neg C$

Conclusion :- $A \to \neg D$

{1}	1.	A	Rule P (additional Premise)
{2}	2.	$A \to B \vee C$	Rule P
{1,2}	3.	$B \vee C$	Rule T $P, P \to Q \Rightarrow Q$.
{1,2}	4.	$\neg C \to B$	Rule P $P \to Q \Leftrightarrow \neg P \vee Q$.
{5}	5.	$D \to \neg C$	Rule P
{1,2,5}	6.	$D \to B$	Rule T $P \to Q, Q \to R \Rightarrow P \to R$.
{7}	7.	$B \to \neg A$	Rule P
{1,2,5,7}	8.	$D \to \neg A$	Rule T $P \to Q, Q \to R \Rightarrow P \to R$
{1,2,5,7}	9.	$A \to \neg D$	Rule T $P \to Q \Leftrightarrow \neg Q \to \neg P$.